Channel Estimation for Orthogonal Time Frequency Space Modulation using Recursive Least Squares

by

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Declaration

I hereby declare that

i) the thesis comprises of my original work towards the degree of Master of Technology in Electronics and Communications at Dhirubhai Ambani Institute of Information and Communication Technology & C.R.Rao Advanced Institute of Applied Mathematics, Statistics and Computer Science, and has not been submitted elsewhere for a degree,

ii) due acknowledgment has been made in the text to all the reference material used.

Bhavesh Amar Singh

Certificate

This is to certify that the thesis work entitled Channel Estimation for Orthogonal Time Frequency Space Modulation using Recursive Least Sqaures has been carried out by Bhavesh Amar Singh for the degree of Master of Technology in Electronics and Communications at *Dhirubhai Ambani Institute of Information and Communication Technology & C.R.Rao Advanced Institute of Applied Mathematics, Statistics and Computer Science* under our supervision.

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Abstract

For its capacity to provide high data rates to a wide number of users, 4G wireless communications had a huge success in the previous decade. With the Internet of Things (IoT) and high mobility scenarios such as vehicle-to-vehicle (V2X) connections on the horizon, the Orthogonal Time Frequency Space (OTFS) modulation scheme has ignited lot of attention in recent years as a viable alternative to OFDM, especially in scenarios involving high user mobility. OTFS has its specialty that it is designed in the delay-Doppler domain. OTFS modulation, when combined with an appropriate equaliser, easily leverages the whole channel variety in both time and frequency. It transforms a fading, time-varying wireless channel used by modulated communications like OFDM into a time-independent channel with a nearly complex channel gain for all symbols. This thesis makes a note on existing drawbacks of OFDM and highlights the usage of a new 2-D modulation scheme called OTFS modulation. It goes on to detail the various methods of channel estimate currently in use while installing OTFS and suggests the use of an adaptive algorithm for channel estimation in the delay-Doppler domain. The proposed algorithm, unlike widely used channel estimation methods, estimates channel gain in the time domain and Doppler taps in the delay-Doppler domain.

Keywords: *5G, OTFS, Delay-Doppler channel, Channel estimations, Adaptive Algorithms, Recursive Lease Squares*

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CHAPTER 1
Introduction

1.1 Motivation

The 21st century has seen a tremendous growth of internet. And with that, it also saw the growth of number of users making use of it. This posed a problem of limiting data bandwidths at that time. And hence it was the hour of need back then to come up with some innovation in the field of communication systems so as to increase the data rates offered to the user. Thus, in the early 2000s, wireless technology gradually shifted from the widely used Wide Band Code Division Multiple Access (WCDMA) to the drastically different emerging technology Orthogonal Frequency Division Multiple Access (OFDMA), which later became the primary focus of 3GPP and was dubbed 3GPP Long Term Evolution or simply LTE.

It has been over 15 years and since then OFDMA has become a central point of wireless technology used in communication systems. With the 4G technology, data rates of up to 300Mbps could be achieved. Along with that, it was also possible to use it in Multiple Input Multiple Output (MIMO) configurations so as to increase the throughput by even more. 4G cellular communications technology also ushered the tremendous growth of smart phones thereby increasing the load of internet usage even more than before. 4G technology changed both the air interface and core network completely.

With a prediction of a greater number of devices to be using internet, somewhere in the middle of previous decade, the term of Internet of Things (IoT) was also coined. Touted as one of the major reasons to bring in Industrial revolution 4.0, IoT envisions a future where most of the machines can be connected via internet for a machine-to-machine level of communication without un-necessary human intervention. This ups the scale of number of user devices which would be using the internet and hence there is a drastic growth in the requirement of bandwidths. This is one of the reasons for companies around the world to look for technology beyond 4G.

Wireless connectivity needs, which are often handled when the user is immobile, are driving today's data rate requirements. In highly mobile environments, however, the performance of currently deployed OFDM systems tends to rapidly decline. Over the past few years, this has led to an interest in the development of 5th-generation cellular communication technologies which are heading into the direction of more carrier investment in order to necessitate new application needs such as Internet-of-Things (IoT) and high-velocity vehicle-to-vehicle communications.

As newer applications are on the emerge, it becomes an area of interest to see if 5G could benefit from a change in modulation and multiple access technique, similar to previous generation jumps from analogue to digital TDMA, CDMA, and OFDM. It has been well established that OFDM achieves high throughput in frequency-selective channels. This optimality, however, is only valid under a set of particular assumptions, including transmitter knowledge of channel state information (CSI), gaussian modulation alphabet, lengthy code-words (implying no latency limits), and complexity on receiver side. Many 5G applications do not stay true to these expectations. For next-generation cellular applications, it is therefore critical to investigate an entirely new modulation system and multiple access design.

Orthogonal Time Frequency Space (OTFS) modulation is a new modulation approach that ensures that every symbol that is transmitted has channel gain that is almost constant, even in channels with strong Doppler or at higher carrier frequencies (mm-wave). One of the most important characteristics of OTFS is that it modulates in the delay-Doppler domain, which is ideal for transmission over time-varying wireless propagation channels.

As a result, OTFS essentially converts the time-varying multipath channel into a 2-D delay-Doppler channel. All symbols in a transmission frame have the same channel gain thanks to this transformation, which is combined with equalisation in this domain.

On the one hand, the delay-Doppler representation of signals can be thought of as a generalisation of the time representation of signals, and on the other hand, as a generalisation of the frequency representation of signals. As a result, OTFS can be thought of as a broadening of OFDM or TDMA. OTFS is a generalisation of (two-dimensional) CDMA since it uses basis functions that span the whole bandwidth and time. In contrast to CDMA and OFDM, the set of OTFS basis functions is designed particularly to tackle the behaviour of a time-varying multipath channel. In a word, OTFS combines the finest aspects of OFDM, TDMA, and CDMA into a single system.

Introduction of OFDM changed the way of air-interface in communication systems. Introducing OTFS would change the way transceivers perform modulation and demodulation in order to extract or encode the data. It therefore becomes imperative to understand the channel characteristics in delay—Doppler domain as well.

Channel estimation has always played an integral part in the process of demodulation and extraction of data from received symbols. Estimating channel state information yields the overall gain the signals have been subjected to while transmission. Channel estimation schemes in currently outgoing 4G systems performs this operation in time-frequency domain. In OTFS, this must be performed in the delay-Doppler domain, since the modulated symbols reside there. Moreover, the delay-Doppler channel representation is the closest representation to a real-world wireless channel geometry. Hence estimating channel in delay-Doppler domain gives a more comprehensive idea of the geometry around the transceivers.

This has led to various algorithm development to estimate the channel state information. This thesis studies through channel properties in delay-Doppler domain and thereafter proposes a novel approach on estimation of channel using adaptive filtering theory.

1.2 Problem Statement

With 5G communication services striving to enable widespread network access, the demand for communication systems capable of delivering high-quality wireless connectivity in high-mobility contexts, such as to and from moving devices, is increasing. The wireless channel in these high mobility situations is doubly dispersive, distributing signals across both time and frequency due to Doppler changes generated by motion in the channel. This is a problem for standard waveforms like OFDM, which suffer from considerable fading in a fast-changing channel.

Recently, the Orthogonal Time Frequency Space (OTFS) modulation scheme was proposed as a solution to these issues. The OTFS waveform's key novelty is that, unlike traditional waveforms that function in the time-frequency domain, symbol multiplexing and detection in the OTFS waveform is done over a grid in the 'delay-Doppler' domain.

The channel response is decomposed into a slowly fluctuating response that closely resembles the physical shape of the channel when viewed in this domain.After demodulation, a delay-Doppler impulse broadcast via the channel appears as a collection of impulses translated according to the constituent propagation pathways' delays and Doppler shifts. Fading is eliminated and the complete diversity of the channel is captured with each sent symbol having the same channel gain if the resolution is sufficient to differentiate the received signal components.. In addition, OTFS can be implemented as a pre- and post-processing stage to standard multicarrier modulation schemes. This allows it to work alongside other 5G technologies

Channel estimation is an important step in a wireless communication system. It allows the transceivers to make better estimate of incoming signals thereby reducing the bit error rate (BER) of the system without compromising on channel capacity or power of the signals during transmission.

The issue of channel estimation for OTFS modulation has been addressed in the recent years. In this thesis, such existing literature on channel estimation for OTFS has been surveyed and mentioned. It has been found that in most practical scenarios, the number of reflectors is significantly very less, which in turn makes the channel sparse in nature. This in turn allows accurate estimation of reflectors. The time-varying nature of the channel allows the use of adaptive algorithms to estimate the channel gain. Hence this led to formation of an adaptive algorithm-based channel estimation scheme for OTFS framework.

1.3 Contribution

Performing detection of OTFS symbols starts with estimating the delay-Doppler channel response and several existing schemes have been explored. In [3] and [4], a pilot-aided channel estimation techniques have been explored. In [5] a pilot OTFS frame has been used for estimating channel information In [4], OTFS channel estimation was conducted in the time–frequency domain. This resulted in higher implementation complexity than that of [3] and [5], where the channel estimation was conducted in delay–Doppler domain. There are several papers focusing on utilizing the sparsity of the channel and they use a compressive sensing approach for estimating the channel [6]. The compressive-sensing-based algorithms are complex to implement and the pilot overhead is significantly larger, sometimes being the complete frame. With a suitable message passing based OTFS detection algorithm [3], the performance of OTFS is in general independent of Doppler frequencies for a given pulse shape unlike OFDM. In [7], probabilistic learning models have been implemented to train the detection algorithm in estimating channel state information. Adaptive algorithms such as Least squares and Recursive least squares have also been utilised in estimating channel state information in convention OFDM systems over different multipath scenarios [8] and variants of RLS have been developed [9] to further enhance their performance metrics.

This thesis addresses the issue of estimating the delay-Doppler channel using adaptive algorithms for OTFS system by making use of RLS algorithm in channel estimation in delay-Doppler domain for an OTFS system.

1.4 Organization of the Thesis

In Chapter 2, we discuss the fundamentals of OFDM systems in brief along with the advantages and disadvantages associated with them. In Chapter 3, basic fundamentals of OTFS modulation, system model and different types of wireless channel representations with their merits and demerits are discussed in detail.

Chapter 4 begins with a brief introduction to adaptive filter theory. Following the introduction, RLS algorithm is discussed. Chapter 5 discusses briefly upon existing literature as part of the literature survey that has been conducted keeping in focus the estimation of DD channel in OTFS modulation systems and developments in adaptive filter algorithms.

Chapter 6 discusses about solution to the problem statement of this thesis. In this chapter, we discuss the algorithm wherein RLS algorithm is clubbed with the pilot-based estimation scheme for estimating the delay-Doppler channel in OTFS scheme. Chapter 7 compares the performance merits of adaptive algorithms over conventional channel estimation algorithms on OFDM systems and then presents the channel estimation results for OTFS system based on the RLS aided pilot-based estimation scheme. Lastly, Chapter 8 discusses the conclusion and future scope in this area.

CHAPTER 2

OFDM (Orthogonal Frequency Division Multiplexing)

2.1 Introduction to Frequency Division Multiplexing and OFDM

The technique of frequency division multiplexing (FDM) divides the overall bandwidth available in a communication medium into a number of non-overlapping frequency bands, each of which carries a separate signal. This lets numerous independent signals to share a single transmission medium, such as a cable or optical fibre. It can also be used to parallelize serial bits or parts of a higher-rate signal.

Traditional modulation methods however became problematic as the need of high data rates kept on climbing. As data rate requirements reached new heights, the symbol duration T_s becomes very small. As a result, the system bandwidth becomes extremely large in order to achieve the data rate.

It is the nature of a wireless channel that it offers a delay dispersion while the signal is transmitted through it. As a result, if the symbol duration is short, the impulse response takes a long time to complete. This in turn also makes the required length of equalizer very long. The computational efforts for such long equalizers is quiet large.

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation method designed for high-data-rate transmission in time-varying settings. It divides a high-rate data stream into several low-rate streams that are sent via parallel, narrowband channels that may easily be equalised.

2.2 Principle of OFDM and Transceiver Design

OFDM is a multi-carrier modulation technique in which information symbols are sent on frequency division multiplexed sub-carriers. These sub-carriers are positioned so that they are orthogonal to one another. Any 2 sub-carries are said to be orthogonal to each other when peak of one subcarrier is located at the zero-crossing points of the remaining sub-carrier.

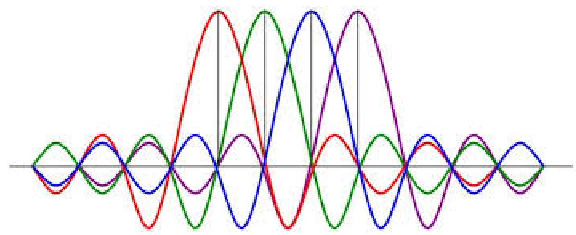

Figure 2.1: Different ways to interpret linear time-varying wireless channels

During the inception of the idea of OFDM, very large number of sub-carrier oscillators were used to implement the frequency division multiplexing operation at transmitter and receiver ends. This is also known as analog implementation of OFDM system. However, later on, usage of Discrete Fourier Transform (DFT) by Weinstein and Ebert was implemented to perform the base-band modulation and demodulation. This eliminated the need of filter banks of sub-carrier oscillators which in turn made the implementation of OFDM system less cumbersome and more efficient. This form is also known as digital implementation of the system.

To begin with the transmission, the data symbols at the transmitter end is first mapped onto the subcarriers in a parallel fashion using Inverse Discrete Fourier Transform (IDFT). This operation is carried out by an IDFT block. While doing so, the data symbols which are present in frequency domain are converted to the time domain for their transmission over the time-varying channel. At the receiver end of the system, the whole operation is performed in inverse fashion, i.e., received data symbols are converted from time domain back to frequency domain by passing them through DFT block. Currently, the IDFT and DFT blocks are replaced by Inverse Fast Fourier Transform (IFFT) and Fast Fourier Transform (FFT) blocks. The OFDM transceivers use an AWGN (Additive White Gaussian Noise) channel to communicate. The system can be used as is, with no modifications, in a frequency-selective channel. As can be observed, the delay dispersion provided by the channel has only a minor impact on OFDM performance.

In fact, OFDM changes a wideband system into a parallel system of narrowband channels, allowing each carrier's symbol duration to be substantially longer than the delay spread. However, when this is not the case, delay dispersion might cause significant errors which come out in the form of interference between subsequent symbols. This type of interference is known as Inter Symbol Interference (ISI). It's also worth noting that delay dispersion causes a loss of orthogonality between subcarriers, resulting in Inter Carrier Interference (ICI).

In order to avoid the ISI, a special type of guard interval known as the cyclic prefix (CP) can eliminate both of these unfavourable effects. The CP is placed as an extension to the existing OFDM symbol in the frequency domain. This is shown in Fig 2.2. Performing this operation converts the transmission of OFDM symbol in time domain as a cyclic convolution.

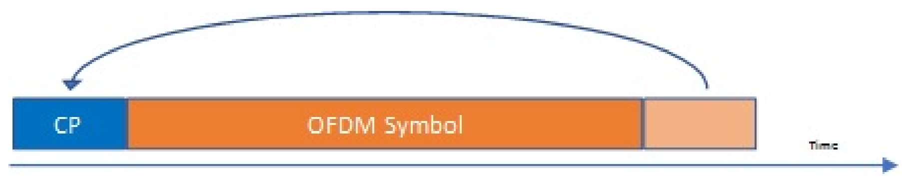

Figure 2.2: Different ways to interpret linear time-varying wireless channels

By introducing the CP, it is possible to retain orthogonality even in poor transmission conditions, removing the effect of ISI. The condition on length of CP is that it should be larger than the maximum delay spread offered by the channel of transmission. CP ensures that the delayed copies of OFDM symbols always have a complete symbol inside the FFT window, resulting in a periodic signal. The system model for OFDM is shown in Fig 2.3. On the transmitter end, an input bit stream of information is generated which is then passed through a suitable modulation scheme. The output from here is then fed to a Serial to Parallel (S/P) block which converts the serial stream into parallel stream which will be fed to the IFFT block. The pilot symbols which act as reference signals for receiver are also added here. By doing the above, it implies that the data is transformed from frequency domain to time domain for transmission purpose. CP is added to this so as to combat the channel impairments. Once the signal is ready in discrete domain, it is converted into its analog counterpart using Digital-to-analog (DAC) converters which is then fed to the transmitting antenna. At the receiver end, once the signal arrives, all operations are done in revere order. First the incoming signal is fed to Analog-to-Digital (ADC) converted which converts it into digital format. Then, the CP removal process takes place. The parallel data is fed to FFT block which then converts the time domain signal into frequency domain for demodulation purpose. Then the streams are sent to demodulation block and finally we get the bit streams.

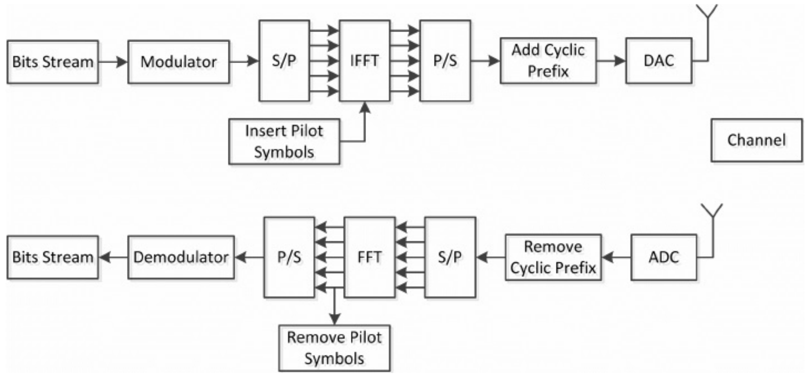

Figure 2.3: Different ways to interpret linear time-varying wireless channels

It is however to be noted that the following conditions are to be kept in consideration:

- the channel is considered to be static for the duration of the OFDM signal in the derivation and interference between the subcarriers can still occur if this assumption is not met.
- the Signal-to-Noise Ratio (SNR) and spectral efficiency are both reduced when some of the received signal is discarded.

2.3 Advantages of design in the transceivers of OFDM compared to previous generations

In summary, the advantages of OFDM scheme compared to previous generations can be listed as:

- Makes efficient use of spectrum by allowing overlap.
- Divide the channel into narrowband flat fading subchannels.
- Eliminates the ISI through cyclic prefix.
- Uses adequate channel coding and interleaving, recovery of symbols can be done for frequency selective channels.
- Channel equalization becomes simpler than by using adaptive equalization techniques with single carrier systems.
- Computationally efficient by usage of FFT techniques.

2.4 OFDM Drawbacks

A typical OFDM system is based on a primary assumption that the time domain channel response is well contained within the CP duration. It is also assumed that the fading in the channel is relatively slow so that the channel can be assumed to be in a static state. However, should the channels be in deep fade, the length of CP in such cases is not able to compensate for the ISI introduced. However, increasing CP also increases the overhead and this affects the spectral efficiency in negative way.

The second major drawback that OFDM suffers also comes from the assumption that the subcarriers are orthogonal. This condition of orthogonality is one of the major pillars for 4G technology. However, it is also possible that in case the subcarriers are not orthogonal, the OFDM system becomes susceptible to interference from frequency offset. This interference is known as Inter-carrier-Interference (ICI).

ICI mitigation techniques have been heavily studied upon and implemented to mitigate this issue. However, with the upcoming high-mobility next generation application scenarios, these techniques do suffer and in turn affect the OFDM systems' performance.

CHAPTER 3

OTFS (Orthogonal Time Frequency Space)

The delay-Doppler formulation of signals can be traced back to findings in physics and maths. P. Bello's seminal paper [10] explains the delay-Doppler representation of time-varying channels in detail, and the generalisation of directional time-varying channels, which is crucial to multiple antenna systems, has also been studied in the past.

Since the 1990s, several papers have proposed for the use of time-frequency diversity transmission, proposing a signal model that presents received signals as a canonical decomposition into delay and Doppler shifted versions on a basis signal, as well as a delay-Doppler RAKE receiver that takes advantage of dispersion in both dimensions. Extensions of these concepts can be found in the use of various training procedures and the use of guard intervals. These works, however, take a different approach than OTFS in that their system designs are in the time-frequency domain rather than the delay-Doppler domain.

This chapter presents the delay-Doppler representation of wireless channels along with a general mathematical description of time-frequency lattice. The findings from this chapter are utilised to explain OTFS modulation in detail in Chapter 4.

3.1 Delay-Doppler Channel

When an electromagnetic wave is transmitted, it experiences a delay in time (delay-shift) and a shift in frequency (Doppler-shift) during its traversal over the wireless channel. Therefore, the receiver receives a delay-Doppler shifted waveform on its end.

The classical paper of Bello [10] makes it clear that time-varying propagation channel can be represented by either a time-varying impulse response, time-varying transfer function, a Doppler-variant transfer function (rarely used) or the Doppler-variant impulse response. Different methods, such as time-frequency, time-delay, and Doppler-delay, can be utilised depending on the parameters used for modelling the response of a linear time changing multi-path channel.

Usually a time-delay $h(t, \tau)$ a time-frequency $H(t, f)$ reresentation are used. However, both these representations are having limitations in that they are characterized by maximum delay and Doppler spreads. The rate of variance of channel coefficients varies (inversely to coherence time) based on the mobility and operating frequency. Because of the increased mobility and operating frequencies, the channel varies fast, making channel estimation problematic. The other

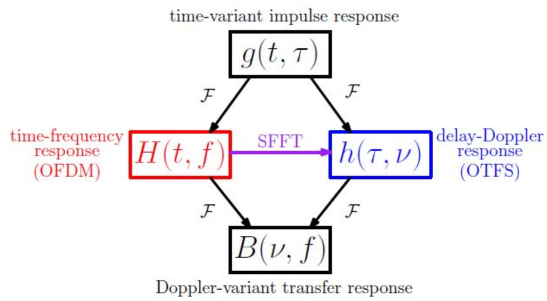

Figure 3.1: Different ways to interpret linear time-varying wireless channels

way of representing the channel is to use delay-Doppler impulse response $h(\tau, \nu)$ where τ and ν are denoting the delay and Doppler respectively. The taps in this domain correspond to the reflectors or group of reflectors having specific delay (depending upon the reflectors' distance) and Doppler values (depending upon the reflectors' velocity). In essence, this depicts the actual geometry of the wireless channel. Because there are only a few reflectors with different delay and Doppler values, this channel representation is small and sparse.

The delay-Doppler taps are time-invariant for a longer observation time than the previous representations because the velocity and distance remain essentially the same over a few milliseconds. This results in estimation of fewer parameters with lesser samples of channel state over larger periods of time thereby bringing down the computational requirements of channel estimation in delay-Doppler domain

Now in the delay-Doppler domain representation, the signal received at the receiver $y(t)$ can be represented as sum of reflected copies of the transmitted signal $x(t)$ which are delayed in time (τ) and shifted in frequency (ν) due to reflectors. Hence this can be represented mathematically by substituting the channel in this domain with the input signal and is given by the double integral as:

$$y(t) = \int_{\nu} \int_{\tau} h_c(\tau, \nu) x(t - \tau) e^{j2\pi\nu(t-\tau)} \, d\tau \, d\nu \tag{3.1}$$

According to (3.1) , the received signal $y(t)$ is a superposition of reflected copies of the transmitted signals, wherein each copy is delayed by a path delay of τ and frequency shifted by the Doppler shift weighed by delay-Doppler response $h(\tau, \nu)$ for that particular delay and Doppler shift. Doppler shifts are typically on the order of 10Hz–1kHz, though this can be rather considerable in the case of exceptionally high mobility or high carrier frequency.

3.2 Time-Frequency Lattice

All time-frequency modulations can be grouped together into an unified framework that includes the following elements:

- A lattice Λ in time-frequency domain that samples the time and frequency axes at integral multiples of T and Δf respectively, that is:

$$\Lambda = (nT, m\Delta f : n, m \in Z) \tag{3.2}$$

- A packet burst lasting NT seconds and having a total bandwidth of $M\Delta f$ Hz.

- A 2D sequence of modulated symbols $X[n, m]$ that we want to transmit over a given packet burst, parameterised along a finite number of points of the lattice Λ with indices $n = 0...N-1$ and $m = 0...M-1$

- A transmit pulse $g_{tx}(t)$ and associated receive pulse $g_{rx}(t)$ whose inner product is bi-orthogonal with respect to translations by integer multiples of time T and frequency Δf, that is

$$\int_t e^{j2\pi m\Delta f(t-nT)} g_{tx}^*(t-nT) g_{rx}(t) dt = \delta(m)\delta(n) \tag{3.3}$$

It is important to note that the bi-orthogonality property in (4) of the pulse shapes ensure that the cross-symbolic interference is eliminated in the symbol reception.

3.3 The Symplectic Finite Fourier Transform

The Symplectic Fourier transform is a variant of 2-dimensaional (2D) Fourier transform which is associated with the Fourier kernel $e^{-j2\pi(m\Delta f - nT\nu)}$ used in for converting between delay-Doppler and time-frequency channel representations.

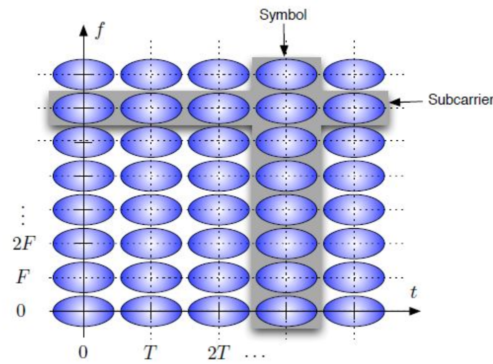

Figure 3.2: 2-D orthogonal plane [1]

Since there is finite number of input signals and frequencies, we shift our focus to the finite version of the transform which is called as Symplectic Finite Fourier transform (SFFT). The input to SFFT is a 2D periodic sequence denoted by $x_p[k,l]$ with periods (M,N) and output of the SFFT is $x_p[n,m]$ with periods (N,M).These input and output sequences should be viewed as defined respectively, along with the points on the time-frequency lattice Λ. The reciprocal delay-Doppler lattice $\Lambda^{\perp}$ that samples the delay axis at integral multiples of $\Delta\tau = \frac{1}{M\Delta f}$,and the Doppler axis at integral multiples of $\Delta\nu = \frac{1}{NT}$, that is :

$$\Lambda^{\perp} = \{(k\Delta\tau, l\Delta v) : k, l \in Z\} \tag{3.4}$$

The delay interval $\Delta\tau$ is inversely proportional to the burst bandwidth $M\Delta f$ and the Doppler interval $\Delta\nu$ is inversely proportional to the burst duration NT Therefore, increasing the bust duration/bandwidth increases the sampling resolution in the delay/Doppler respectively. This is consistent with radar principles, which state that the range/velocity resolution of probing waveforms is proportional to their bandwidth/duration. The output sequence is given by the following formula:

$$\boldsymbol{X}_p[n,m] = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} x_p[k,l]e^{-j2\pi\left(\frac{mk}{M}-\frac{nl}{N}\right)} \tag{3.5}$$

With a minus sign, the SFFT couples the frequency variable with the delay variable and the time variable with the Doppler variable. Symplectic coupling is the name given to this sort of coupling. The inverse transform $x_p[k,l] = SFFT^{-1}$ $x_p[n,m]$ which can be written as follows:

$$x_p[k,l] = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} \boldsymbol{X}_p[n,m]e^{j2\pi\left(\frac{mk}{M}-\frac{nl}{N}\right)} \tag{3.6}$$

The interchangeability of circular convolution and point wise multiplication of periodic sequences is a property of the SFFT, which is equivalent to the convolution property of the standard finite Fourier transform.

3.4 OTFS Fundamentals

It has been observed that although OFDM modulation scheme has proven itself to be highly sought-after scheme due to its robustness and simplicity in implementation as well as achieving high data rates, it is also a fact that OFDM modulation scheme suffers a heavy performance degradation in high Doppler-sensitive environmental conditions. In this chapter, a recently developed modulation technique is discussed which is Orthogonal Frequency Time Space (OTFS) modulation technique. It is essentially a result of attempts to combine the governing principles of CDMA and OFDMA technologies and to bring out the best of both. It combines the principle of spread spectrum technique which provides resilience to narrow band interference and the principle of orthogonality among the subcarriers which simplifies the channel coupling so as to achieve high throughput with lower complexity and high performance. Unlike the above-mentioned schemes (CDMA and OFDMA), OTFS modulation scheme operates on the delay-Doppler model of the channel. It has been established in the previous chapters that delay-Doppler channel model is able to map the exact geometry of the wireless channel compared to the other channel model representations. This can be picturised as shown in the following Fig 3.3. OTFS modulation makes use of processing signals in the delay Doppler domain so as to take the advantage of the channel model. To make it clearer, it must be looked upon into the basic signal representation, both in time as well as frequency domain. A time domain representation of signal can be realised as superposition of delta functions whereas the same signal in frequency domain can be realised as superposition of complex exponentials. It is also established fact that the two representations are interchangeably used by making use of Fourier transforms. In the previous chapter, this complementary nature of time and frequency representation has been captured while explaining the Heisenberg's uncertainty principle. To reiterate, Heisenberg's uncertainty principle states that a signal can't be localized in both of the domains (i.e., time and frequency) simultaneously. This means that if a signal is time-localized, it is frequency-localized, and conversely.

Signals that behave as though they are localised in both domains (time and frequency) at the same time do exist. In a form known as the delay-Doppler rep-

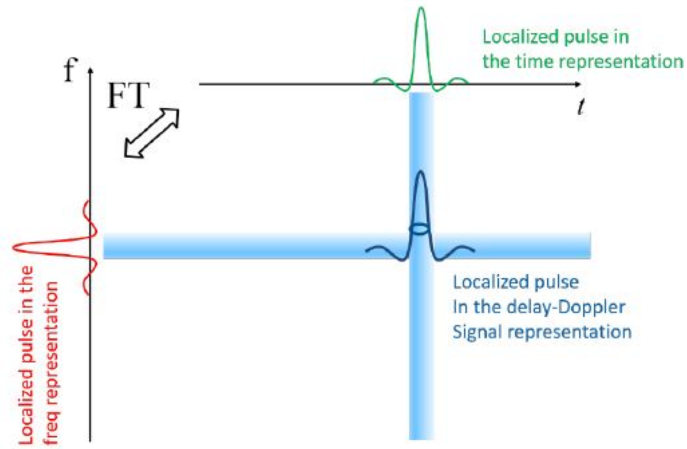

Figure 3.3: Time and Frequency representation of signals [2]

resentation, these signals are linked to localised pulses. Such signals are characterised by the delay and Doppler variables in the 2D domain known as the delay-Doppler domain. As discussed in previous chapters, the wireless channel in this domain is represented by time and frequency shift operations.To start with, a delay-Doppler signal is defined as a function which satisfies the following condition:

$$\phi\left(\tau + n\tau_r, v + mv_r\right) = ej2\pi\left(nv\tau_r - m\tau v_r\right)\phi(\tau, v) \tag{3.7}$$

where τ_r is the delay period and ν_r is the Doppler period satisfying the condition $\tau_r.\nu_r = 1$.

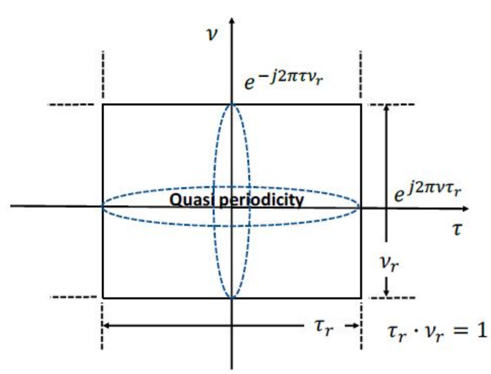

Figure 3.4: Quasi Periodicity in Delay-Doppler domain [2]

This is also known as the quasi-periodic condition, in which the value of the function acquires a phase factor of $e^{j2\pi\tau v_r}$ for each traversal of delay period τ_r and, reciprocally, acquires a phase factor of $e^{j2\pi\tau v_r}$ for each traversal of Doppler period ν_r. And hence it can be seen that a signal can be represented in three fundamental ways, first way is as a function of time, second as a function of frequency and lastly as a quasi-periodic function of delay and Doppler. The conversion between time and frequency is done by the means of Fourier transform similarly conversion between delay-Doppler representation to other representations is done by Zac transforms Z_t and Z_f.

$$Z_t(\phi) = \int_0^{v_r} e^{j2\pi tv}\phi(t, v)dv \tag{3.8}$$

$$Z_f(\phi) = \int_0^{\tau_r} e^{-j2\pi tv}\phi(\tau, f)d\tau \tag{3.9}$$

It's important to note that the Zak transform must satisfy the quasi-periodicity condition in order to be a one-to-one equivalence between 1D line and 2D delay-Doppler plane functions. Without the periodic requirement, the delay-Doppler representation of the signal on the 1D line will have an endless number of representations.This is related to the Fourier equivalence between sampled functions on the line and periodic functions on the line. A sampled function will have an infinite number of representations in the Fourier domain if periodicity is not imposed.

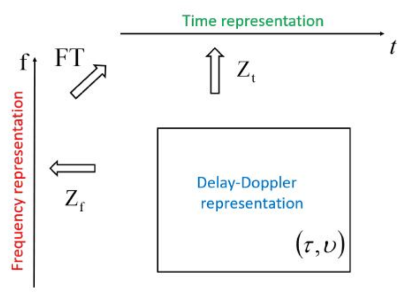

Figure 3.5: Delay-Doppler representation of signal [2]

The relationship shared between these 3 domains can be summarized as shown in the following fig 3.6 Each edge of the triangle can be represented as a transformation and it can be seen that transformation on one edge can be written as composition of the pair of transformations on the other two edges of the triangle. For example, a Fourier transform can be written as composition of two Zak transforms. This implies that instead of using the Fourier transform for transforming from frequency to time domain, one can use inverse Zak transform for transformation from frequency to delay-Doppler domain and then use Zak transform for transformation from delay-Doppler domain to time domain. It is also to be noted that that the delay-Doppler representation is not unique but depends on the choice of the pair of (τ_r ,ν_r) satisfying the relation $\tau_r \cdot \nu_r = 1$ This corre-

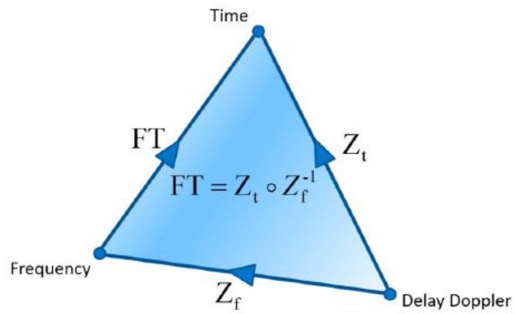

Figure 3.6: The fundamental triangle [2]

sponds to a large family of delay-Doppler representations based on the hyperbola $\nu_r = 1/\tau_r$. It is worth notign as to what happens as we approach the limits when the variable $\tau_\mathrm{r} \to \infty$ and when the variable $\nu_\mathrm{r} \to \infty$. The delay period is extended at the expense of the Doppler period contracting in the first limiting case, resulting in a one-dimensional representation that coincides with the time representation in the limit. The Doppler period is extended at the expense of the delay period contracting in the second limiting case, resulting in a one-dimensional representation that coincides with the frequency representation in the limit. As a result, the time and frequency representations can be thought of as limiting examples of the larger delay-Doppler representations family. By using correctly designed Zak transformations that satisfy commutativity connections generalising the triangle relation stated earlier, all delay-Doppler representations can be interchanged. This means that any pair of representations along the curve can be converted regardless of whatever polygonal path is used to connect them.

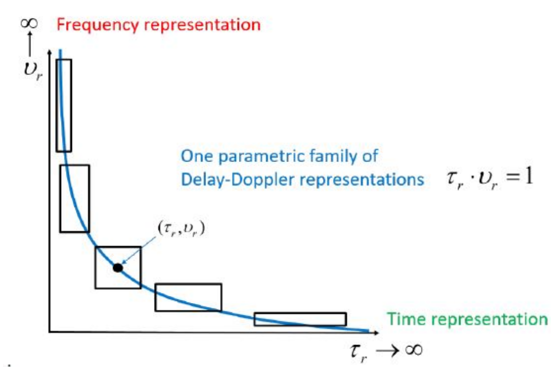

Figure 3.7: Parametric representation of delay-Doppler domain [2]

Based on above mentioned points, it is observable that converting the Time Domain Multiplexed (TDM) pulse to the delay-Doppler representation results in quasi-periodic function which is localized in the delay domain but it is non localized in the Doppler domain. Similarly, it is the opposite case when Frequency

Domain Multiplexed (FDM) pulse is converted to the delay-Doppler representation which results in quasi-periodic function which is localized in the Doppler domain but non-localized in the delay domain. As a result, as illustrated in the diagram below, OTFS is a modulation method based on symmetrically localised signals in the delay-Doppler representation.

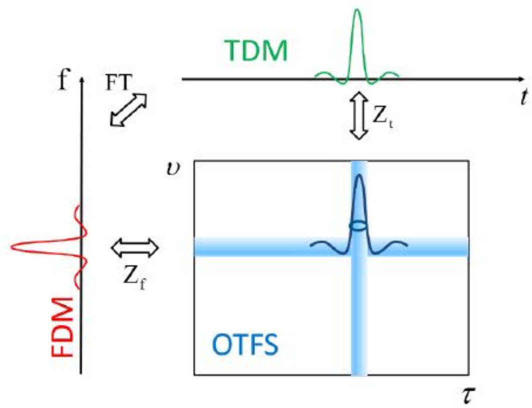

Figure 3.8: The relationship between TDMA, FDMA and OTFS [2]

As discussed in the previous chapter, a wireless channel can be modelled as the collection of reflectors which can either be stationary or moving. Because of the relative velocity between the reflector and the transmitter/receiver, the transmitted wave is reflected by a reflector with a frequency shift, and these reflections arrive at the receiver with a delay. There will also be a change in the amplitude based on the constructive or destructive interference due to numerous reflectors sharing the same properties of delay and Doppler. The different ways in which channel effects the three signals discussed above i.e.the TDM, FDM and the OTFS waveforms can be visualised by following example.

By transmitting a localized TDM pulse we can separate the reflections of each reflector based on the delays but when reflectors have same delay but different Dopplers they are superimposed on each other and hence cannot be separated. This can be seen where the TDM reflections from left to right, the first and third reflections are time invariant as they are stationary, the last reflection is time variant as its moving and lastly the second reflection is the superimposition of the two reflectors where one is stationary while the other is moving. By transmitting FDM pulse we can separate the reflections of each reflector based on the Doppler values but when reflectors have same Dopplers but different delays they are superimposed on each other and hence can't be separated. This can be seen where

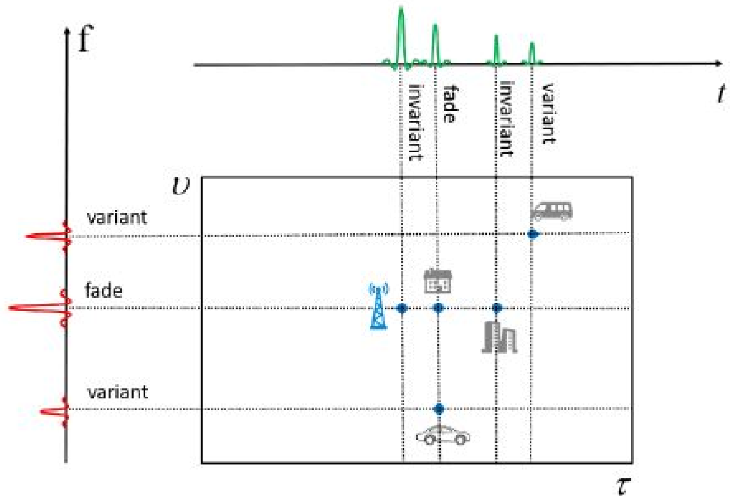

Figure 3.9: TDM and FDM channel interactions. [2]

considering the FDM reflections from top to bottom the first and last reflections have different frequencies as they are moving at different velocities. The middle reflection is due to the superposition of the static reflectors.

We obtain reflections with particular delay-Doppler shifts generated by different reflectors when we transmit a localised OTFS pulse in the delay-Doppler channel, as seen in the above figure. The phase and amplitude of the delay-Doppler reflections are unaffected by the originating pulse's domain position. These reflections are also easily separated based on the delay and Doppler values, hence there is no interference and no loss of energy. These reflections are also orthogonal to each other. OTFS is a time-frequency spreading scheme that consists of a set of two-dimensional basis functions specified over a time-frequency grid.

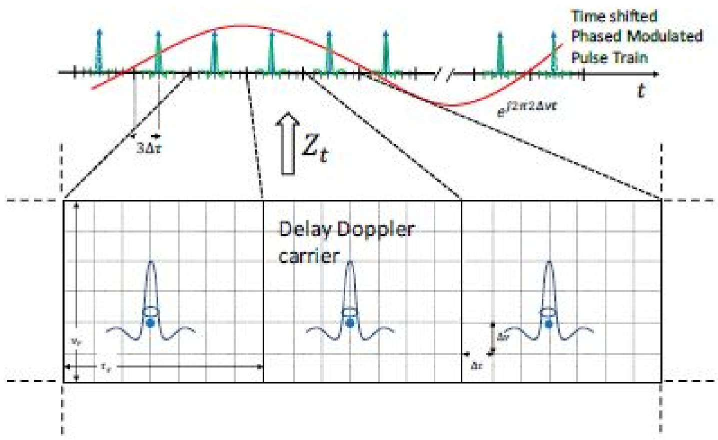

Figure 3.10: OTFS signal in delay-Doppler grid. [2]

Another example of a delay-Doppler channel and time-frequency channel is shown in below figures. This is a 10-tap channel (10 prominent reflectors in the setup) and the simulation has been created on MATLAB environment.

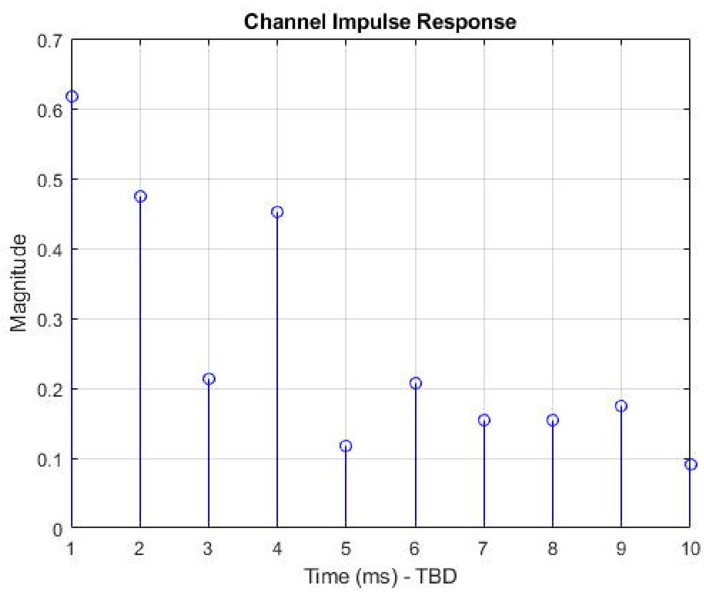

Figure 3.11: Channel Response in Time-domain

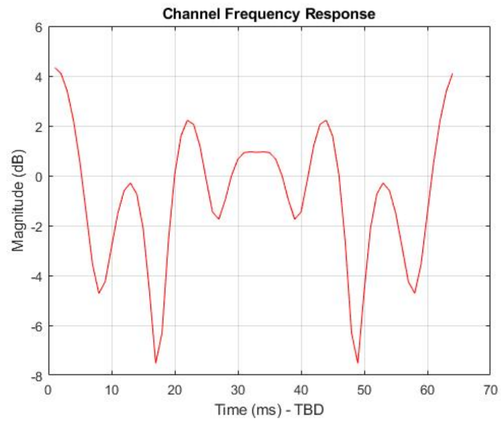

Figure 3.12: Channel response in Frequency domain

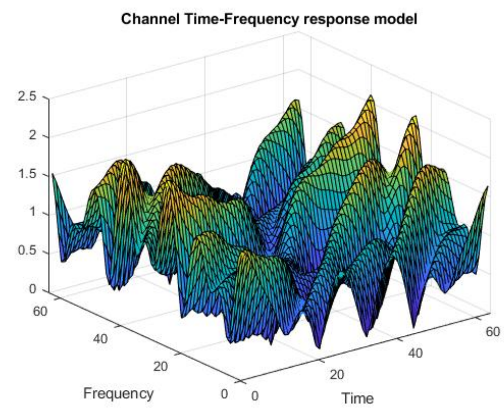

Figure 3.13: Time-frequency channel representation of 10-tap channel.

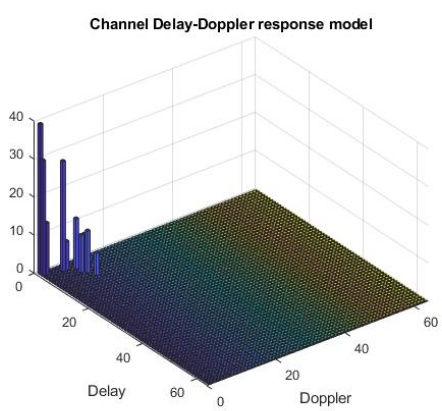

Figure 3.14: Delay-Doppler Channel representation of the 10-tap channel.

It can be seen that the delay-Doppler channel representation is quite sparse compared to the highly varying time-frequency channel suffering from Doppler.

OTFS is designed as a pre-processing block for the multicarrier modulation schemes like the OFDM. This is based on the duality of the Fourier duality between the delay-Doppler and the time frequency grids.

The delay Doppler grid consists of M points along the delay axis of spacing of

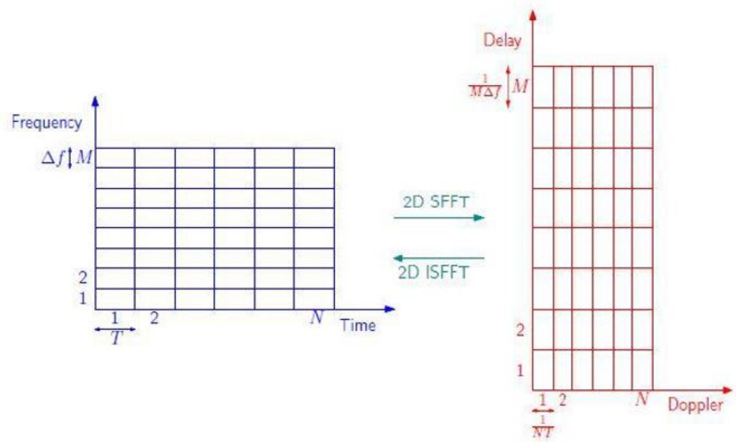

Figure 3.15: Delay-Doppler and Time-Frequency grids.

$\Delta\tau = \tau r M$ and N points along the Doppler axis with spacing of $\Delta v = v_r/N$ and the time-frequency grid consists of M points along the frequency axis with spacing $\Delta f = 1/\tau_r$ and N points along the time axis with a spacing $\Delta t = 1/\nu_r$. The time frequency grid can be interpreted as a sequence of N multicarrier symbols each having M subcarriers. The parameter Δt is the multicarrier symbol duration and the parameter Δf is the subcarrier spacing. It is to note that the bandwidth of the transmission $B = M\Delta f$ is inversely proportional to the delay resolution $\Delta\tau$. and the duration of the transmission $T = M\Delta t$ is inversely proportional to the Doppler resolution $\Delta\tau$. Both the grids are presented in the above figure. The Fourier relation between the two grids is a variant of the 2D FFT called as Simplectic Finite Fourier transform (SFFT). SFFT transforms the time frequency domain into the delay-Doppler domain or the inverse simplectic finite Fourier transform (ISFFT) transforms the delay-Doppler domain to time-frequency domain which is shown in the below equation.

$$X[n,m] = \frac{1}{MN}\sum_{k=0}^{N-1}\sum_{l=0}^{M-1} x[k,l]e^{j2\pi\frac{nk}{N}-\frac{ml}{M}} \tag{3.10}$$

ISFFT can be seen as applying an M-dimensional FFT along the columns of the delay-Doppler matrix x[k ,l] followed by applying N-dimensional IFFT along the rows of the matrix to get the time-frequency domain X[n, m] .

3.5 OTFS System Model

One of the ways to implement OTFS modulation is using the pre- and post-processing blocks in existing OFDM scheme systems. This also adds to the ease of transitioning to this scheme in future. The block diagram for OTFS modulation is shown in Fig.3. Here the information symbols are represented by $x_p[k, l]$ (which can be QAM/QPSK symbols) residing in the delay-Doppler domain. They are first mapped to the familiar time-frequency domain symbols X p [n,m] using 2D Inverse Symplectic Finite Fourier Transform (ISFFT) and windowing. Together, they form what is called OTFS transform.

Each element in $x_p[k, l]$ modulates a 2D basis function that completely spans the transmission time and bandwidth in the time-frequency domain. The OFDM transform (IFFT) which is represented here by a generalized Heisenberg transform is then applied to the time-frequency transformed symbols $x_p[n, m]$ to convert the time-domain signal $x(t)$ for transmission.

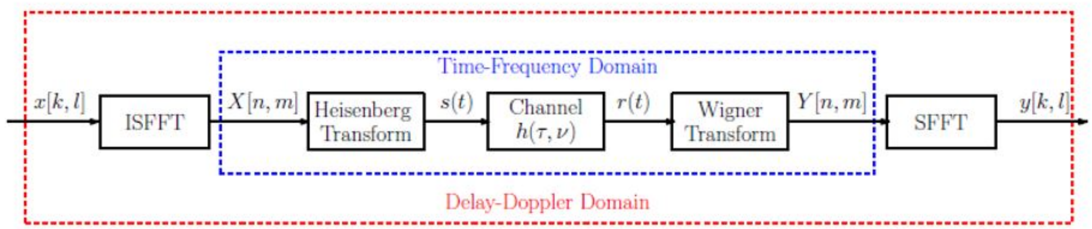

Figure 3.16: OTFS Modulation Block diagram

The received signal $y(t)$ is converted back to time-frequency domain by Wigner transform, which is generalization of inverse OFDM transform (FFT). Post this,$Y[n, m]$ is transformed to the delay- Doppler signal $y[k, l]$ through Symplectic Finite Fourier Transform (SFFT). The aforementioned explanation, as well as the block diagram, show that OTFS modulation is a scheme that adds additional pre- and post-processing to a multi-carrier system that uses time-frequency domain representation (OFDM). The mathematical formulations to execute the above flow diagram are :

1. **Inverse Symplectic Finite Fourier transform (ISFFT)**: There are MN information symbols which are multiplexed on a delay-Doppler grid size of $N \times M$ These symbols on the delay-Doppler domain are denoted by $x[k, l]k = 0, 1, ...N - 1, l = 0, 1..., M - 1, x[k, l] \in A$ where A is a conventional modulation alphabet (for eg. QAM), are transmitted in a packet duration of NT

in a given bandwidth of $B = M\Delta f$,where $\Delta f = \frac{1}{T}$.These symbols in delay-Doppler domain are first mapped to the time-frequency (TF) plane using ISFFT, as follows:

$$X[n,m] = \frac{1}{MN}\sum_{k=0}^{N-1}\sum_{l=0}^{M-1} x[k,l]e^{j2\pi\frac{nk}{N}-\frac{ml}{M}} \tag{3.11}$$

2. **Modulator**: The TF signal $X[n,m]$ is then converted to the time domain signal for transmission using Heisenberg's transform:

$$x(t) = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} X[n,m]g_{tx}(t-nT)e^{j2\pi m\Delta f(t-nT)} \tag{3.12}$$

where $g_{tx}(t-nT)$ is the transmit pulse shape. This simplifies to IFFT in case of N = 1 and rectangular g_{tx} (OFDM block)

3. **Channel**: The time domain signal is then transmitted through the time varying channel, whose complex based band delay-Doppler response is denoted by $h(\tau,\nu)$, where τ and ν denote the delay and Doppler respectively. The received time-domain signal is given by:

$$y(t) = \iint h(\tau,\nu)x(t-\tau)e^{j2\pi\nu(t-\tau)}\,d\tau\,d\nu \tag{3.13}$$

4. **Demodulator**: The received signal $y(t)$ is converted into a time-frequency signal using Wigner Transform:

$$Y(t,f) = A_{grx}[t,f] = \int g^{*}_{yx}((t\prime - t)e^{j2\pi(t\prime - t))}\,dt \tag{3.14}$$

$$Y[n,m] = Y(t,f)|_{t=nt,f=m} \tag{3.15}$$

where $g_{rx}(t)$ denotes the receive pulse shape. This simplifies to FFT in case of N = 1 and rectangular g_{tx} (OFDM block).

5. **Symplectic Finite Fourier Transform (SFFT)** : The TF signal $Y[n,m]$ is transformed back to the delay-Doppler domain using SFFT, as

$$X[n,m] = \frac{1}{MN}\sum_{n=0}^{N-1}\sum_{m=0}^{M-1} x[k,l]e^{j2\pi(\frac{nk}{N}-\frac{ml}{M})} \tag{3.16}$$

A perfect localization in time and frequency of g_{tx} and g_{rx} makes them satisfy the condition of bi-orthogonality.

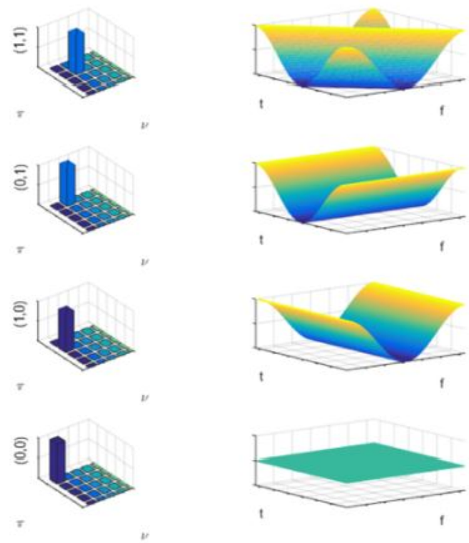

Figure 3.17: Symplectic Fourier dual basis functions in the time-frequency domain (right) and corresponding 2D basis functions in the delay-Doppler domain (left)

In case of perfect localization, the time frequency input-output relation can be shown as

$$Y[n,m] = H[n,m]X[n,m] \tag{3.17}$$

where

$$H[n,m] = \sum_{l}\sum_{k} h[k,l]e^{j2\pi(\frac{nk}{N}-\frac{ml}{M})} \tag{3.18}$$

Based on the system model shown in Fig.5, we have developed following equations so as to realize the OTFS model. We have the information symbols in X, to transmit the signal in 2D time domain we perform the operations: ISFFT, OFDM modulation (IFFT), Pulse shaping in combination to obtain the symbol matrix S:

$$S = G_{tx}{F_M}^H F_M X {F_N}^H \tag{3.19}$$

Eq (3.19) simplifies further to

$$S = G_{tx} X {F_N}^H \tag{3.20}$$

Similarly, on the receiver end, we have received symbols in R which can be represented in delay-Doppler domain by performing the operations of pulse shaping, OFDM demodulation (FFT), SFFT to obtain the received symbols in Y

$$Y = F_M{}^H F_M G_{rx} R F_N \tag{3.21}$$

Eq (3.21) simplifies further to

$$Y = G_{rx} R F_N \tag{3.22}$$

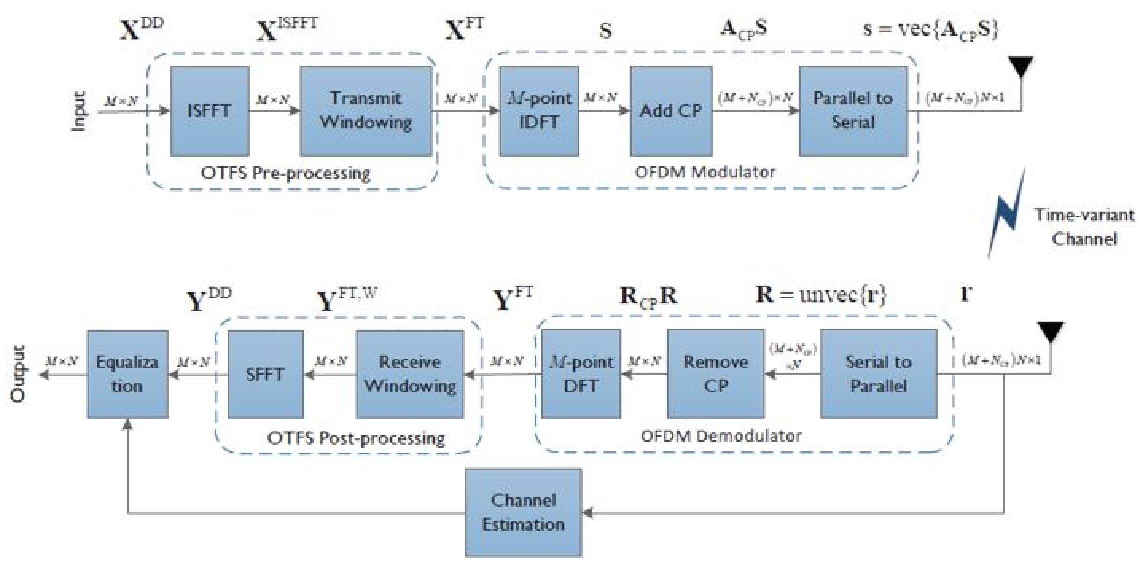

Figure 3.18: OTFS SISO architecture

The above figure is a general representation of a SISO-OTFS architecture which includes all the steps that we have discussed up till now.

CHAPTER 4
Adaptive Filter Theory

In signal processing, filters play a major role in extracting meaningful information from incoming data which is heavily corrupted by noise. In the context of communication systems, this noise creeps into the data streams via various causes led by channel distortion. And hence the data received at the receiver end is termed as noisy data and the first step on receiver end is to remove the noise and estimate the original signal or data which was intended to be sent by the transmitted.
Filters can be of both types – linear and non-linear. However due to computational complexity reasons, linear filters are preferred more so. Again, within linear filters, there have been many which have been developed in the past and are still used today such as wiener filters, Kalman's filters where the main idea of the algorithms resides on reducing the mean square error value. Iterative channel estimation methods have been explored and investigated by using these algorithms in case of SC-FDMA. However, the algorithms in this makes an assumption that there is some information about channel state available prior to the operation of estimation, for eg, noise correlation. This poses as a limitation in case where CSI is not available to the user. Hence to mitigate this, the approach can be modified by making use of adaptive filters. An adaptive filter as the name suggests works on recursive algorithm by updating system parameters (which are under observation, in this case information related to channel – channel taps) using a gradient-based method. The idea is to update the parameter until certain prefixed error criteria is matched and then the system stops updating. To begin with this, we consider the error reduction problem by a cost function. In this case, the cost function is considered as mean-squared-value of error (difference between the desired response and the output of the filter). The updating of parameters takes place by considering the present set of parameter values and gradient based parameter (learning parameter) over previous parameters. This concept can be

expressed as follows:

$$h(n+1) = h(n) + \text{ correction term} \tag{4.1}$$

where correction term is based on the error, input signal and a learning rate. One of the commonly used adaptive algorithm with above problem statements is Recursive Least Squares (RLS) algorithm (Appendix 1.). RLS algorithm among provides a faster convergence compared to rest of the commonly used adaptive algorithms such as Least mean squares (LMS), Normalised Least Mean Squares (NLMS). RLS is an adaptive filter algorithm that recursively finds the coefficients that minimize the weights of linear least squares cost functions relating to the input signals. This approach falls in contrast with other adaptive algorithms such as Least mean squares (LMS) that aim towards reducing the mean square error.

4.1 Recursive Least Squares Algorithm

Adaptive algorithms are ones which change behaviour with time. This change is based on current information as well as some pre-defined criterions based on previously available information. It represents a system of linear filters wherein we have a transfer function which is controlled by varying parameters as per an optimization algorithm.
RLS algorithm recursively finds the coefficients that minimize a weighted linear Least Square (LS) cost function which is related to the input signal sequence.

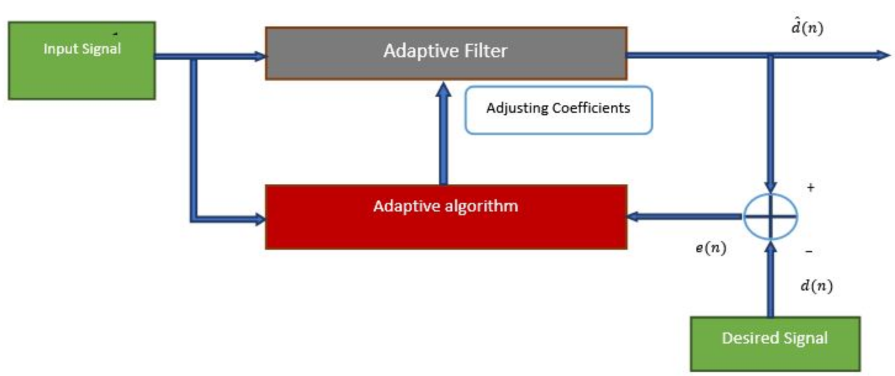

Figure 4.1: Adaptive Filter Block Diagram

In a linear LS estimation, the observed data is represented as,

$$X = A\theta + e \tag{4.2}$$

where A is assumed to be a full-rank matrix, θ is the parameter vector and e is a zero-mean random vector. Components of e are uncorrelated and have same variance.
LS estimation minimizes the sum square error (SSE) by using the following cost function:

$$J(\theta) = (X - A\theta)'(X - A\theta) \tag{4.3}$$

In this case, the LS estimator is given by,

$$\widehat{\theta}_{LS} = \left(A'A\right)^{-1} A'X \tag{4.4}$$

In the case of RLS algorithm, we instead have a weighted SSE and for this we perform the following operations:
$h(n)$ constitutes of filter coefficients at an instant . The cost function for the algorithm is given by

$$J(n) = \sum_{k=0}^{n} \lambda^{n-k} e^2(k) \tag{4.5}$$

which simplifies to

$$J(n) = \sum_{k=0}^{n} \lambda^{n-k} \left(d(k) - y'(k)h(n)\right)^2 \tag{4.6}$$

Here $e(k)$ is the filtering error at the instant k due to $h(n)$ and $textstyle\lambda$ is the forgetting factor which lies between 0 and 1. To solve the problem of LS estimation, we minimize $J(n)$, with respect to $h(n)$. This is given by:

$$\frac{\partial J(n)}{\partial h(n)} = 0 \tag{4.7}$$

Substituting $J(n)$ and $h(n)$,gives the following normalized equation for LS estimation:

$$\left(\sum_{k=0}^{n} \lambda^{n-k} y(k)y'(k)\right) h(n) = \sum_{k=0}^{n} \lambda^{n-k} d(k)y(k) \tag{4.8}$$

Let us define

$$\hat{R}_y(n) = \left(\sum_{k=0}^{n} \lambda^{n-k} y(k)y'(k)\right) \tag{4.9}$$

which is the estimator of autocorrelation matrix Ry. Similarly, we can define

$$\hat{r}_{dy}(n) = \sum_{k=0}^{n} \lambda^{n-k} d(k) y(k) \tag{4.10}$$

which is the estimator of cross-corelation vector $\hat{r}_{dy}(n)$.Substituting Eq. (4.10) & (4.9) in (4.8), we can re-write (4.8) as

$$\widehat{R}_y(n) h(n) = \hat{r}_{dy}(n) \tag{4.11}$$

The solution to Eq. (4.11) comes as:

$$h(n) = \widehat{R}_y^{-1}(n) \hat{r}_{dy}(n) \tag{4.12}$$

The matrix inversion in Eq. (27) makes the solution a computationally complex algorithm. This prompted us to look for a recursive solution to the above problem. To find the inverse, we can represent

$$\hat{R}_{dy}(n)$$

in recursive way. It can be re-written as

$$\widehat{R}_y(n) = \lambda \widehat{R}_y(n-1) + y(n) y'(n) \tag{4.13}$$

Eq. (28) shows that that autocorrelation matrix can be recursively calculated from its previous values and present data vector. Similarly, the cross-corelation vector is given as

$$\hat{r}_{dy}(n) = \lambda \hat{r}_{dy}(n-1) + d(n) y(n) \tag{4.14}$$

The solution now can be re-written as

$$h(n) = \left(\lambda \hat{R}_y(n-1) + y(n) y'(n)\right)^{-1} \hat{r}_{dy}(n) \tag{4.15}$$

To calculate the inverse of the sum matrix, we make use of matrix inversion lemma:

$$\left(A + UV'\right)^{-1} = A^{-1} - \frac{A^{-1} U V' A^{-1}}{1 + V' A^{-1} U} \tag{4.16}$$

- Provided that

$$1 + V'^{A^{-1}} U \neq 0$$

- In this case, once we know A^{-1} is known, the inverse of rank-one increment

$$\left(A+UV'\right)^{-1}$$

can easily be calculated.

To calculate $\hat{R}_y^{-1}(n)$, we will take the matrix lemma inversion with

$$A=\lambda\widehat{R}_y^{-1}(n-1) \text{ and } U=V=y(n) \tag{4.17}$$

Substituting the above, we have,

$$\begin{aligned} \hat{R}_y^{-1}(n) &= \left(\lambda\hat{R}_y(n-1)+y(n)y'(n)\right)^{-1} \\ = \frac{1}{\lambda}&\left(\hat{R}_y^{-1}(n-1)-\frac{\hat{R}_y^{-1}(n-1)y(n)y'(n)\hat{R}_y^{-1}(n-1)}{\lambda+y'(n)\widehat{R}_y^{-1}(n-1)y(n)}\right) \end{aligned} \tag{4.18}$$

Let $P(n) = \hat{R}_y^{-1}(n)$. Substituting this, we have,

$$P(n)=(\frac{1}{\lambda})\left(P(n-1)-\frac{P(n-1)y(n)y'(n)P(n-1)}{\lambda+y'(n)P(n-1)y(n)}\right) \tag{4.19}$$

Simplifying (31) we can write this more neatly as

$$P(n)=(\frac{1}{\lambda})(P(n-1)-k(n)y'(n)P(n-1)) \tag{4.20}$$

where , $k(n)=\frac{P(n-1)y(n)}{\lambda+y\prime(n)P(n-1)y(n)}$, $[k(n) isGainFactor]$

Using the gain factor equation, we can arrive at the following result:

$$k(n)\lambda+k(n)y'(n)P(n-1)y(n)=P(n-1)y(n)y'(n)P(n-1) \tag{4.21}$$

$k(n)$ is an important parameter for adaptation. It is also related to the current data vector $y(n)$ by

$$k(n)=P(n)y(n) \tag{4.22}$$

Substituting the above relations in the original filter update equation, we can arrive at the following result:

$$h(n)=h(n-1)+k(n)(d(n)-y'(n)h(n-1)) \tag{4.23}$$

CHAPTER 5

Existing solutions for Channel Estimation in OTFS scheme

To perform detection of OTFS symbols on receiver end, the first step is to estimate the delay-Doppler channel response at the receiver. There are several existing schemes that have been applied on this problem.

In [3],[4],[11] pilot-aided channel estimation techniques have been explored. The paper [3] proposes a Markov Chain Monte Carlo (MCMC) based algorithm for detection and a pseudo-random noise (PN) pilot-based scheme for channel estimation in the delay-Doppler domain. The authors make use of randomized Gibb's sampling-based detection algorithm in order to do so. In [4], pilot-aided channel estimation was considered for OTFS with ideal pulse-shaping waveform over channels with integer Doppler shifts only, i.e., when the channel Doppler taps are aligned to integer delay–Doppler grid. Further, the exact symbol deployment and channel estimation technique are not described in [4].

In [5], a complete OTFS frame has been used as a pilot for transmission and the estimated channel information from this was further used for data detection in the subsequent frame. One of the key highlights of this method is that the pilot overhead required for CSI estimation is very high. Furthermore, this method may not prove to be effective if the channel estimation becomes outdated in the following frame, as there is an assumption that the channel response will be static at least for the upcoming frames of data. Similarly, in [12], the concept of estimating channel using a complete OTFS frame has been extended to MIMO-OTFS setup and the author establish a 3-D structured sparsity of MIMO-OTFS channel and make use of a 3D-SOMP algorithm which is an extension of OMP algorithm to estimate the channel. However, the complexity of the algorithm proposed in here is high and is based on efficient channel feedback which may or may not be the case always.

In [4], OTFS channel estimation was conducted in the time–frequency domain. This resulted in higher implementation complexity than that of [3] and [5], [12] where the channel estimation was conducted in delay–Doppler domain.

There has also been keen focusing on utilizing the sparsity of the channel and making use of a compressive sensing approach for estimating the channel [6]. The compressive-sensing-based algorithms are complex and the pilot overhead is significantly larger, sometimes being the complete frame. With a suitable message passing based OTFS detection algorithm as in [3], the performance of OTFS is in general independent of Doppler frequencies for a given pulse shape unlike OFDM.

In [13], the channel estimation scheme proposed makes use of priori channel information based on multiple impulses so as to make use of diversity gain and come up with more accuracy than traditionally proposed schemes in [3] [4]. However, this scheme relies on priori channel state information which may not be available in the initial stage of device's communications.

In [7], the author makes use of sparse CSI estimation model where the pilots are directly transmitted over the TF-domain grid. This results in a significantly reduced pilot overhead and an increased bandwidth efficiency. The author then developed a Bayesian-learning (BL) framework. The DD-domain CSI obtained was then fed to the BL framework in order to estimate the subsequent frames of data detection. The associated BL framework however requires an initial training time to train the model in order to produce effective results.

In [9], it was found that channel sparsity aware version of RLS algorithms have been deployed to non-linear systems and active noise control problem areas. This helps in reducing the complexity by using area specific data selection sequences. In [8], a comparative study was conducted by the authors on adaptive algorithms – LS, RLS for estimating channel in different multipath fading environments. The differences between the two algorithms and merits and demerits along with performances have been presented for reference in this paper.

CHAPTER 6

Pilot-based Estimation scheme

Based on the previous sections, the problem of delay-Doppler channel estimation has 2 parts: estimating the position of reflectors and estimating the magnitude of gain provided by these reflectors. The position of the reflectors is estimated by observing the received signal frame in delay-Doppler domain. We arrange a pilot at the centre of the delay-Doppler grid on the 1st frame on the transmitter side. This pilot symbol has considerably higher power compared to rest of the information symbols in the grid. The pilot symbol is also surrounded by guard symbols from all sides so as to contain the delay and Doppler spread while traversing through the time-varying channel. Following fig. describes one such arrangement of pilot symbol frame.

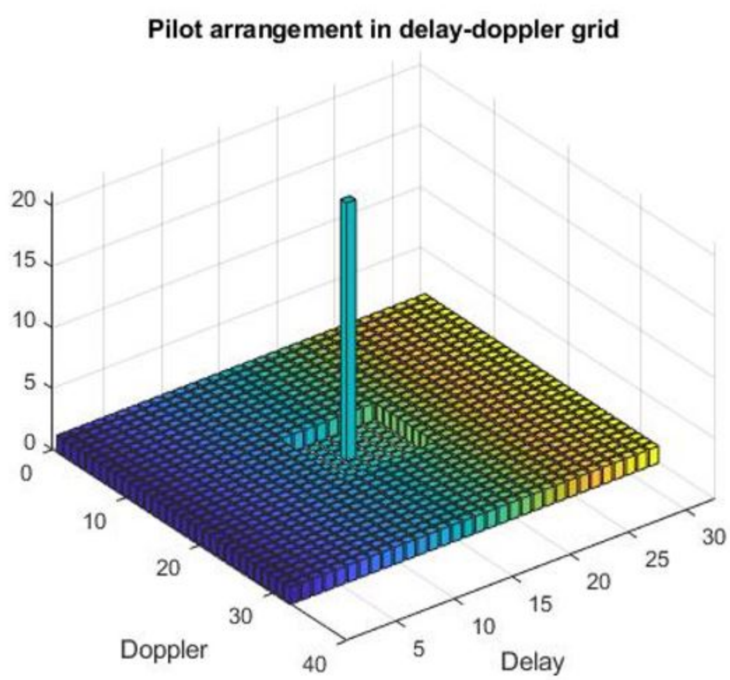

Figure 6.1: Pilot placement on delay-Doppler grid

The magnitude of channel taps is estimated with making use of RLS algorithm has been approached as shown in the flowchart below.

This transmitting symbol when moved from delay-Doppler domain to the time-frequency domain are embedded with a preamble pilot at the start of the OFDM frame. In this way, the first symbol contains a pilot preamble which is used to estimate the magnitude for channel taps at the receiver using RLS algorithm. Once both, position and magnitude are acquired, the channel estimate in the delay-Doppler domain can be formed. This channel is of M x N dimension.

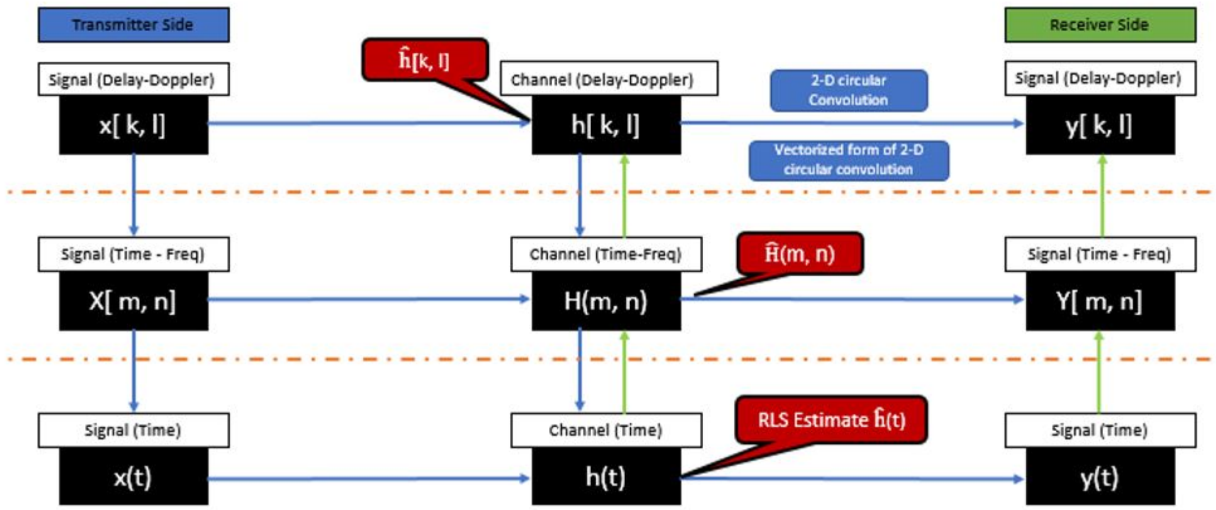

Figure 6.2: Workflow of Channel estimation problem

CHAPTER 7
Results

7.1 OFDM Performance with RLS-CE

To test the adaptive algorithm, an OFDM system simulation was made where receiver uses adaptive algorithms to estimate the channel and then perform error calculation on the received signal vector. In this case, the information was passed using QPSK modulation schemes before passing the symbols on OFDM transmitter. The channel estimation done in this case was using preamble-based pilot estimation, wherein a pilot OFDM symbol was attached towards the start of the frame. The receiver performs RLS algorithm-based channel estimation using the pilot preamble. The performance is found to improve slightly over the conventional LS estimation technique.

7.2 Channel estimation in OTFS system

Following the channel estimation technique described in previous chapters, the position of reflectors are estimated from the received signal frame in delay-Doppler domain. The delay-Doppler grid in consideration is of size 32 x 32. The pilot is placed at (16,16) with a guard band surrounding it of the size 7 x 7. Fig.6 showcases this arrangement. The received pilot and guard frame in delay-Doppler domain are shown in following figure

RLS algorithm was used to estimate the magnitude gain provided by the channel in time-domain channel. The simulation parameters for the results are summarized in the following table:
From Fig.12 & 13, it can be seen that the channel estimate in delay-Doppler domain is very similar to the original channel along with perfect recovery of position of reflectors and the respective gains that they provide.

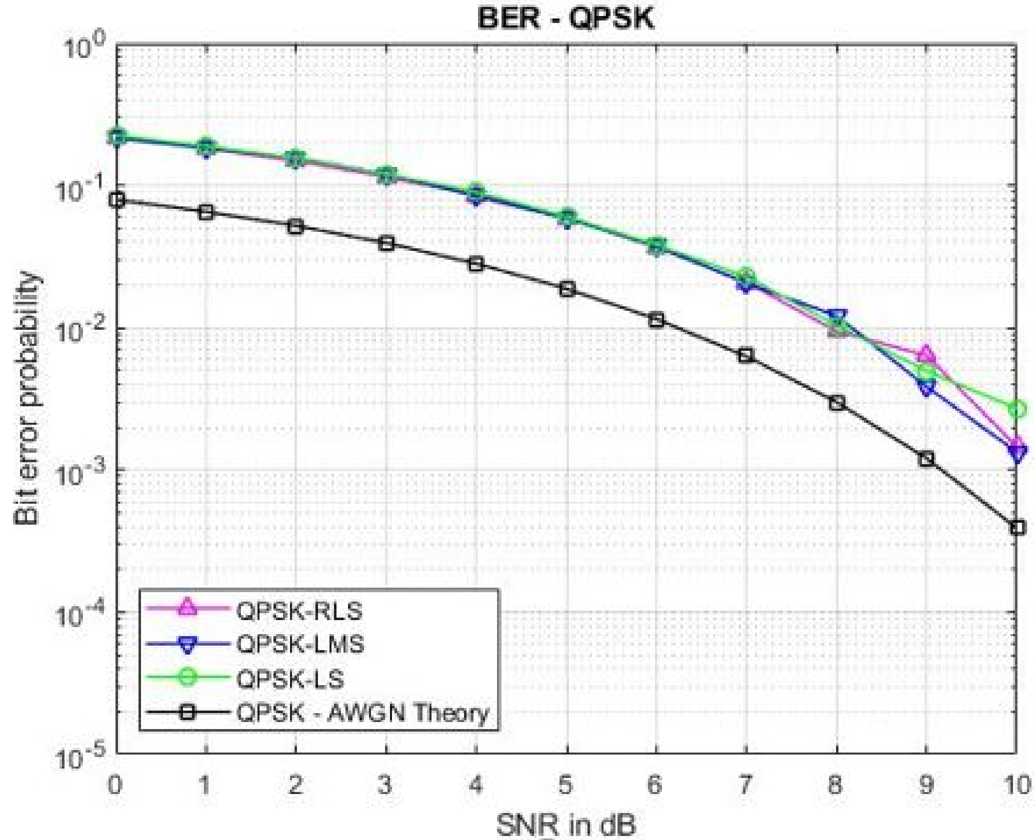

Figure 7.1: Performance of RLS, LMS & LS in OFDM system for QPSK modulation.

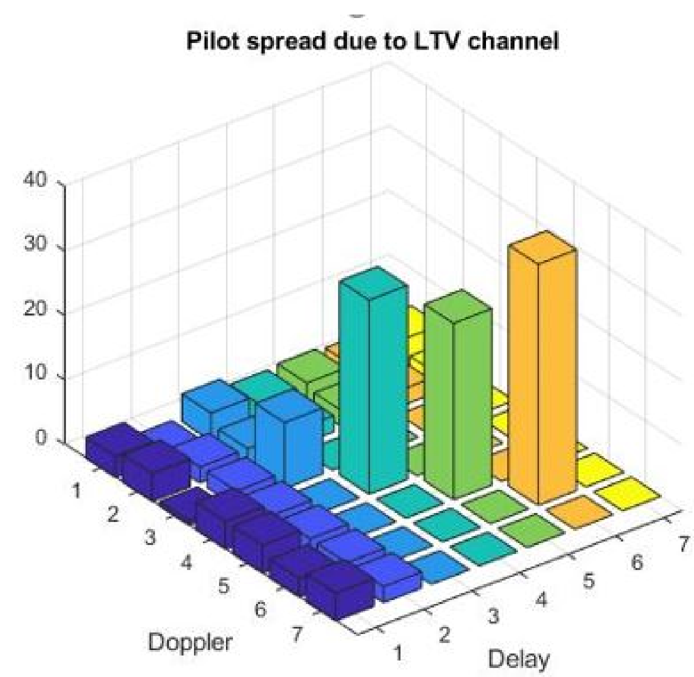

Figure 7.2: Spread of Pilot within guard band due to LTV channel

M	32
N	32
No of channel taps	4
No of frames	1
No of samples used for RLS estimation	32
Forgetting factor λ	0.99

Table 7.1: Table 1

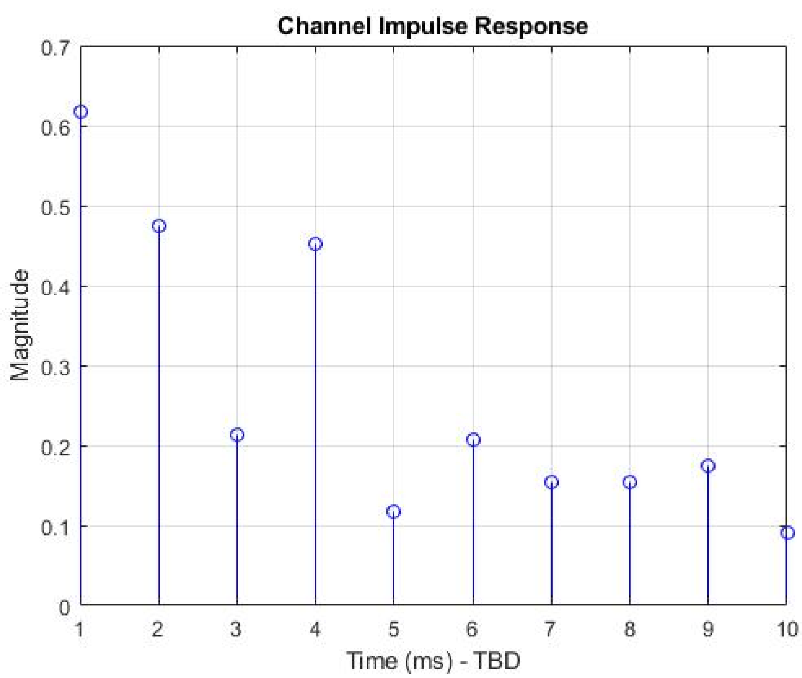

Figure 7.3: Channel estimate in time-domain

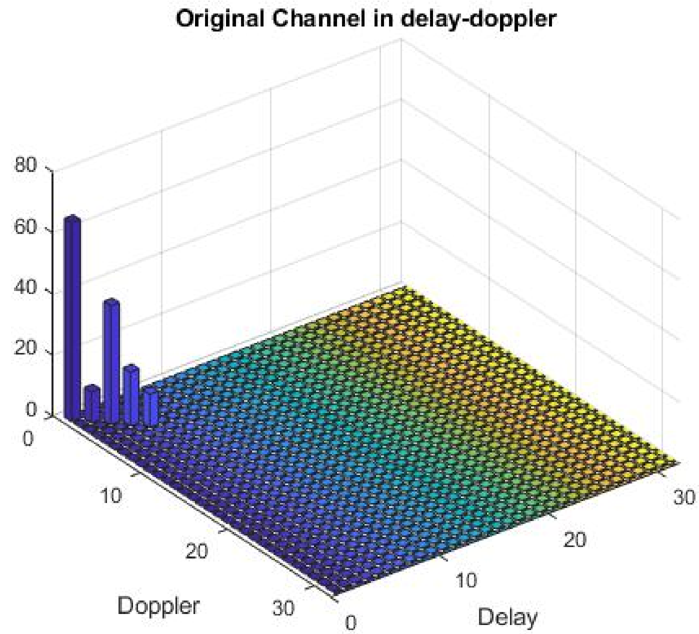

Figure 7.4: Original Channel in Delay-Doppler domain

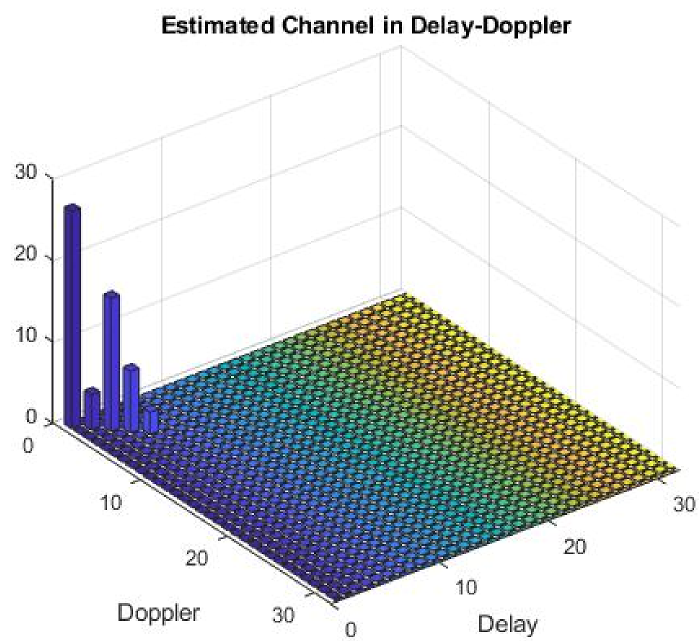

Figure 7.5: Estimated Channel in Delay-Doppler domain

7.3 Study of 2-D adaptive algorithms to estimate 2-D delay-Doppler channel for OTFS

Study on 2-D adaptive algorithms had been made for OTFS systems. In OTFS delay-Doppler channel, there are two parameters of estimation – delay and Doppler. An attempt to further tailor the RLS algorithm in 2D has also been looked upon wherein delay and Doppler coefficients for channel can be estimated simultaneously. Currently, work is in progress with respect to making use of 2-D algorithms in delay-Doppler domain.

CHAPTER 8

Conclusions and future scope

8.1 Conclusions

Fig 6.1 shows the performance of an OFDM system wherein the BER performance of the system is observed. In this system, a bit stream modulated with QPSK symbols was transmitted and at the receiver end, Least squares, RLS algorithms were applied to retrieve the information.The computational time of RLS algorithms is much faster than LS algorithm and yet it delivers a performance which was in comparable range of LS. This established the fact of faster convergence of RLS algorithm without suffering losses in precision of estimation.

Moving ahead, we create an OTFS frame with pilot at the centre and surrounded by the guard band. This frame when transmitted on a LTV channel experiences spread both in delay and Doppler. This is observe and captured in Fig 6.2. next we create a delay-Doppler channel in OTFS system and place our pilot in the delay-Doppler grid as shown in Fig 6.2. Since the channel used in this case is a M x N channel where grid size is at 32 x 32 with 4 channel taps. The channel's time response is estimated and compared with original channel time response magnitudes, which is depicted in Fig 6.3.

Fig 6.4 Fig 6.5 are channel responses in delay-Doppler domain capturing the Doppler estimates of the LTV channel in consideration. It can be seen that both figures have exactly same position with comparable magnitudes of each Doppler coefficient.

Hence , the magnitude of the channel response and the exact position of Doppler variables is estimated correctly.

8.2 Future Scope

The current implementation of RLS algorithm as discussed in Chapter 5, has been explored when the system in consideration is a SISO-OTFS model. This opens up ground for exploring other adaptive algorithms within the same problem frame.

This can also be extended to a MIMO-OTFS system model with different adaptive algorithms and the results can be compared on the metrics of complexity, energy and resource consumption and resulting estimates of channel state information.

As mentioned in chapter 6, development of a 2-D based adaptive algorithm is also in progress which can further improve the performance of the systems to estimate a dynamic DD channel.

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