

Identification of Block-Sparse Systems using Adaptive Filtering Algorithms

by

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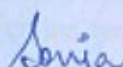


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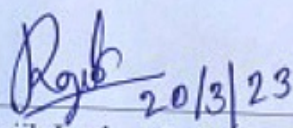
- i) the thesis comprises of my original work towards the degree of Master of Technology in Electronics and Communications at Dhirubhai Ambani Institute of Information and Communication Technology & C.R.Rao Advanced Institute of Applied Mathematics, Statistics and Computer Science, and has not been submitted elsewhere for a degree,
- ii) due acknowledgment has been made in the text to all the reference material used.



Sonia

Certificate

This is to certify that the thesis work entitled Identification of Block-Sparse Systems using Adaptive Filtering Algorithms has been carried out by Sonia for the degree of Master of Technology in Electronics and Communications at *Dhirubhai Ambani Institute of Information and Communication Technology & C.R.Rao Advanced Institute of Applied Mathematics, Statistics and Computer Science* under our supervision.



Rajib Lochan Das
Thesis Supervisor

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Rajib Lochan Das
Thesis Supervisor

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Abstract

An adaptive filter is a system with a linear filter that has a transfer function controlled by variable parameters and a means to adjust those parameters according to an optimization algorithm. Adaptive filters are used for linear time-variant systems where the characteristics of the systems keep on changing with time. Therefore, adaptive filters are required for some applications when some parameters of the desired processing operation are not known in advance or are changing.

In the world of adaptive algorithms, sparse system identification has received a lot of interest. In numerous applications, including acoustic echo cancellation, interference reduction in industrial settings, and biomedical engineering, system identification is regularly encountered. During the last ten years, system identification has been widely used in a variety of signal processing applications, including wireless communication, radar imaging, and echo cancellation.

A sparse impulse response is one in which a significant portion of the energy or information is concentrated in a few number of its impulse response coefficients. There are few non-zero or high coefficients and numerous tap-weights with zero or tiny values in various cases, such as network echo cancellation, where the impulse responses are sparse. Sparse systems come in a variety of forms. The conventional one is referred to as a block-sparse system, like TV transmission channels. The non-zero coefficients of block-sparse systems consist of one or more clusters, and a cluster is a set of non-zero or big coefficients, in contrast to generic impulse response sparse systems where large coefficients are distributed at random. This thesis has taken into consideration various existing adaptive algorithms, viz, LMS, NLMS, PNLMS, ZA-NLMS, ZA-PNLMS, BS-PNLMS, BS-IPNLMS to identify a block-sparse system with the help of mean square error and the convergence rate of the coefficients. It continues to give a proposed algorithm with some modifications to get a better convergence rate for the coefficients of an unknown system which is assumed to be a block-sparse system for our research.

Keywords: *Adaptive filters, Adaptive Algorithms, System identification, Sparse systems, Block-sparse systems, Mean square error, Convergence rate*

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CHAPTER 1

Introduction

1.1 Motivation

The discipline of digital signal processing has advanced significantly during the past three decades. The discipline of digital signal processing has had such rapid growth and development that several of its specialized issues have grown into other fields. The adaptive filter is an illustration of a digital signal processing system. The goal of filtering is to process a signal with the intention of manipulating the information it contains.

An adaptive filter's ability to self-modify based on real-time input and its iterative approach have led to widespread use in a variety of issues, including system identification, channel estimation, echo cancellation, and many more. Numerous adaptive algorithms have been created to solve these issues and arrive at the best solution based on a certain optimization criterion. A digital filter is one that decodes digitally encoded signals. It is simple to choose the best algorithm to handle a signal is when working with signals whose statistical features are fixed. However, if a signal's qualities are unknown, fixed algorithms cannot process it effectively. In such cases, utilizing an adaptive filter that may adjust its properties automatically is the remedy. The transfer function of the closed loop adaptive filter is improved via feedback in the form of an error signal. In general, the closed loop adaptive process uses a cost function, a measure of the filter's performance, to feed an algorithm that decides how to change the filter transfer function to reduce costs on the following iteration. The mean square of the error signal is the most prevalent cost function.

In many cases, it is possible to anticipate that the impulse response of unidentified systems will be sparse, with a sparse distribution of large and small coefficients. A sparse signal is one that contains only a few relatively large amplitudes while the rest are zero or very small. The filtering performance can be improved by using such sparse prior data. The standard adaptive filters, however, do not make use of this data. Numerous adaptive algorithms have been created to solve these issues and arrive at the best solution based on a certain optimization criterion. The least mean square (LMS) algorithm and its variations are among the most well-liked techniques that have been widely applied for this purpose among these algorithms. For situations where the systems are sparse in nature, these approaches have low convergence rates, high power consumption, and are sensitive to strongly correlated input.

Due to its wide range of applications, sparse signal recovery has recently gained popularity in the field of signal estimation. This motivates the development of better adaptive filters with enhanced convergence behaviour and error cost function for the sparse signal recovery problem. Such a sparse structure in systems can be found in many real world applications such as digital TV transmission, acoustic echo channel and system identification.

In practical scenario of system identification, the unknown system may be sparse. For the purpose of recognizing sparse systems, several algorithms have recently been developed. The impulse response of a family of sparse systems, block-sparse systems, consists of one or more clusters and a cluster is a collection of non-zero coefficients. Examples of block-sparse models are the acoustic echo path and satellite-linked or indoor MIMO communication systems. During the past few years, many algorithms exploiting sparsity and block-sparsity have been developed. Many variants take into consideration proportional step sizes to different taps or coefficients according to their magnitudes, such as the proportionate normalized least-mean square (PNLMS) algorithm and its variants. The recent progress in block-sparse systems identification has motivated to use the l_0, l_1 -norm or mixed norms, $l_{1,2}$ norms penalty to exploit sparsity or block-sparsity. The basic idea is to introduce a penalty that favors sparsity in the cost function.

1.2 Problem Statement

In the world of adaptive algorithms, sparse system identification has received a lot of interest. The family of proportional algorithms, which are widely employed in network echo cancellation (NEC), etc., employs the sparseness of a given system to enhance the convergence performance of normalised least mean square (NLMS).

The proportionate NLMS (PNLMS) aims to independently update each filter coefficient by changing the adaptation step size in proportion to the predicted filter coefficient [1]. For example, when the echo path is sparse, the PNLMS, as opposed to the NLMS, has very quick initial convergence and tracking. Although the big coefficients converge quickly (fast initial convergence), the convergence of the small coefficients (after the initial period) is significantly slowed down [2],[3]. The remaining small coefficients adapt more slowly than NLMS as the large taps do.

Recently to enhance the identification of block-sparse systems, a block-sparsity-induced adaptive filter known as block-sparse LMS (BS-LMS) has been developed [4]. The fundamental idea behind BS-LMS is to modify the cost function of the conventional LMS algorithm by adding a block-sparsity penalty (a mixed $l_{2,0}$ norm of adaptive tap-weights with equal group partition sizes). A group of proportionate algorithms for block-sparse system identification is proposed. These algorithms can

converge more quickly in block-sparse applications. The suggested approach includes both the traditional NLMS and the PNLMS algorithms as special examples.

The another recent proposal on block-sparse system employs the use of mixed $l_{2,1}$ norm in place of l_0 or l_1 norm for computing the gain as compared to conventional PNLMS algorithm, which is called as the BS-PNLMS [5]. In this thesis following such existing algorithms and literature, another method or algorithm has been tried to be developed which will provide better and faster convergence for block-sparse system identification.

1.3 Contribution

In this thesis,

- System identification of block-sparse systems is performed with estimating the optimum weight coefficients and establishing better convergence rate for active taps or the larger weight coefficients.
- Several existing adaptive algorithms have been studied and explored.
- Many research papers focus on utilizing the sparsity of the system and signal by using a compressive sensing approach to estimate the weights or coefficients [6], [7]. However, the compressive-sensing-based algorithms are complex to implement [8].
- There are conventional system identification methods, such as the LMS and its variants, which give poor performance in terms of steady state excess mean square error (EMSE) and convergence speed because they do not employ the a-priori knowledge of sparsity of the system.
- Further, the existing algorithms which can harness the sparsity, viz, PNLMS, ZA-PNLMS [9], are employed for identifying sparse systems. These existing algorithms use proportionate idea to estimate or optimize the coefficients.
- A new family of proportionate normalized least mean square (PNLMS) adaptive algorithms that improve the performance of identifying block-sparse systems is proposed.
- The main proposed algorithm, called improved block-sparse PNLMS (IBS-PNLMS), is based on the optimization of a mixed $l_{2,1}$ norm of the adaptive filter coefficients.
- Here we address the issue of obtaining better convergence rate of coefficients of unknown system, assumed to be block-sparse system, using adaptive algorithms, for sparse and block-sparse systems, and by employing BS-PNLMS and IBS-PNLMS algorithms for system identification.

1.4 Organization of the Thesis

The structure of the thesis is organized in the following order.

In Chapter 2, we discuss the fundamentals of adaptive filters in brief along with the applications of their filtering action. We also discuss the use of LMS, NLMS and PNLMS algorithm in adaptive filtering for system identification in brief.

In Chapter 3, sparsity and the existing algorithms to harness sparsity are discussed in detail.

In Chapter 4 we discuss the proposed algorithm for block-sparse system identification.

Chapter 5 discusses about solution to the problem statement of this thesis. In this chapter, we discuss the algorithm wherein IBS-PNLMS algorithm is modified to obtain better convergence rate of coefficients. Lastly, Chapter 6 discusses the conclusion and future scope in this area.

CHAPTER 2

Adaptive Filter Theory

This chapter gives a review of adaptive filtering. At first a brief about the motivation for using adaptive filters is introduced, then we discuss some popular applications of adaptive filters. Finally, some of the most commonly used adaptive algorithms are introduced.

2.1 Adaptive filtering

Numerous practical signal processing applications cannot be solved satisfactorily using fixed digital filters. This is because the full range of input conditions may not be known precisely or even statistically, and because the design criteria may change during the normal operation of the filter. The majority of these applications are amenable to being successfully resolved by use of a unique class of filters called adaptive filters. The ability of adaptive filters to adjust their response in response to periodic variations in the input statistics sets them apart from other filter types. Due to its capability, it is a potent tool for signal processing and control applications. Numerous industries, including communications, radar, sonar, seismology, and bio-medical engineering, have effectively used adaptive filters.

Adaptive filters are self-adjusting and time-varying systems. An adaptive filter can track the changes in input statistics over time and perform satisfactorily in an unknown environment. The four fundamental groups of adaptive filtering applications are identification, inverse modeling, prediction, and interference cancellation [10]. Channel Equalization is a type of adaptive filtering that belongs to the Inverse Modeling class. An adaptive equalizer's job, given a channel with an uncertain impulse response, is to alter the channel output in such a way that the channel and equalizer, when connected in cascade, approximate a perfect transmission medium [11]. Without utilizing the precise statistical characteristics of the pertinent signals, the adaptive channel equalizer seeks to mimic the performance of the optimal filter.

An adaptive filter's primary goal is to configure its parameters so that the output attempts to minimize a significant objective function involving a reference signal. The input, reference, and output signals of the adaptive filter are often functions of the goal function F . The definitions of the minimization algorithm, the objective function, and the error signal can be thought of as the three fundamental components that make up an adaptive algorithm. The difference between an intended response and the filter output is typically used to define the error signal. The ideal filter parameters are determined by minimizing a cost function of the error signal. A practical strategy relies on minimizing the error signal's mean-square value [10].

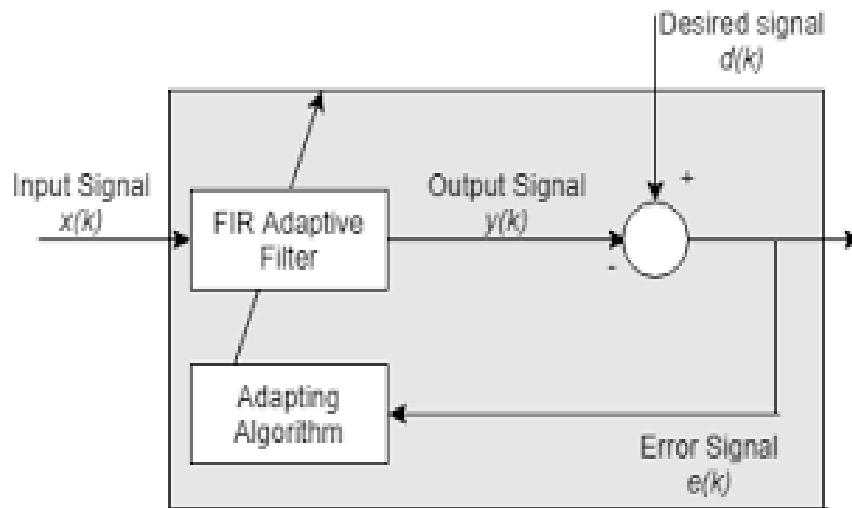


Figure 2.1: The basic structure of an adaptive filter. After [12].

For convenience of presentation, the following notation is used in the figure:

$x(k)$ = input applied to the adaptive filter;

$y(k)$ = output of the adaptive filter;

$d(k)$ = desired response;

$e(k) = d(k) - y(k)$ = estimation error.

The adaptation method then uses the error signal to create a performance function, which is iteratively reduced to decide how to update the filter coefficients. The objective function's minimization suggests that the output signal of the adaptive filter somewhat resembles the desired signal [15].

The two main classes of adaptive digital filter are essentially the finite-duration impulse response (FIR) filter and the infinite-duration impulse response (IIR) filter, which are distinguished by the nature of the impulse response. IIR filters use recursive structures, whereas FIR filters are implemented with non-recursive structures. The most widely used adaptive filters are FIR filters because of their stability. The mostly used type of adaptive FIR filter is the transversal filter. The basic structure of a FIR filter is shown in figure 2.2.

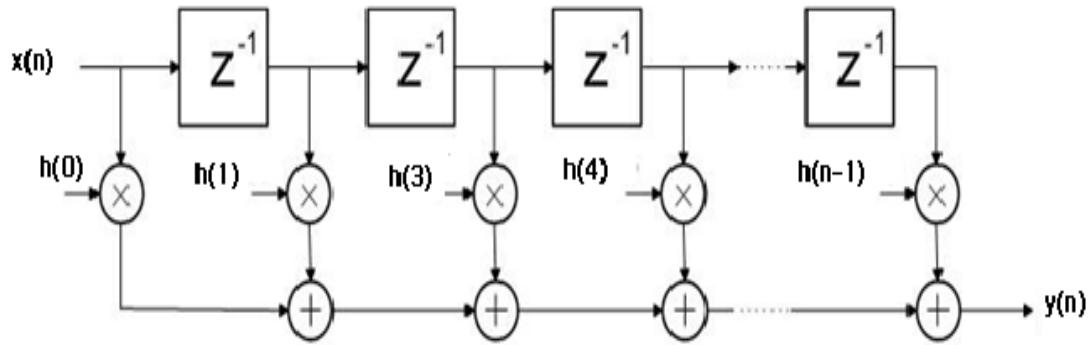


Figure 2.2: Basic structure of FIR filter. After [13].

The FIR filter is chosen over the IIR filter because it is intrinsically stable due to the use of just forward paths in its structure, whereas the IIR filter uses both feed-forward and feedback paths. FIR filters are utilized in most applications because the adaptive filters with feedback are more challenging to manage. The transversal or a tapped delay line FIR filter is shown in figure 2.2 is used in many implementations of adaptive filters.

The three fundamental components which constitute the filter are a unit-delay element, a multiplier, and an adder. The amount of delay elements employed in the filter determines its order. The multiplier's function is to multiply the input from the tap to which it is connected by a filter coefficient known as the tap weight. The adders' combined function in the filter is to add the outputs of each separate multiplier to get an overall filter output that is

$$y(k) = \sum_{i=0}^{N-1} h(i) \cdot x(k - i) \quad (2.1)$$

In Table 2.1, some applications of adaptive filtering are briefly given.

Class of adaptive filtering	Application
I: Identification	System identification Layered earth modeling
II: Inverse modeling	Predictive de-convolution Adaptive equalization
III: Prediction	Linear predictive coding Signal detection
IV: Interference canceling	Echo Cancelation Adaptive beam forming

Table 2.1: Applications of adaptive filtering

In this thesis, system identification is focused and discussed.

2.1.1 System identification application

Science and engineering both depend on the concept of a mathematical model. An adaptive filter is used to generate a linear model that, in some ways, reflects the best match to an unknown plant in the class of applications dealing with identification. The identical input powers both the plant and the adaptive filter. The desired response for the adaptive filter is provided by the plant output. The model will change over time if the plant is dynamic.

System identification is the experimental approach to the modeling of a process or a plant. Planning the experiment, choosing a model structure, estimating the parameters, and validating the model are the steps involved. Here, we briefly describe the concept of adaptive filtering techniques for predicting the parameters of an unidentified plant modeled as a transversal filter. Let's say we have an unknown linear time-varying plant. This plant can be determined by a set of discrete-time measurements that show how the output changes in response to a known input. A finite number of unit-delay elements constitute the model, along with a corresponding set of adjustable parameters.

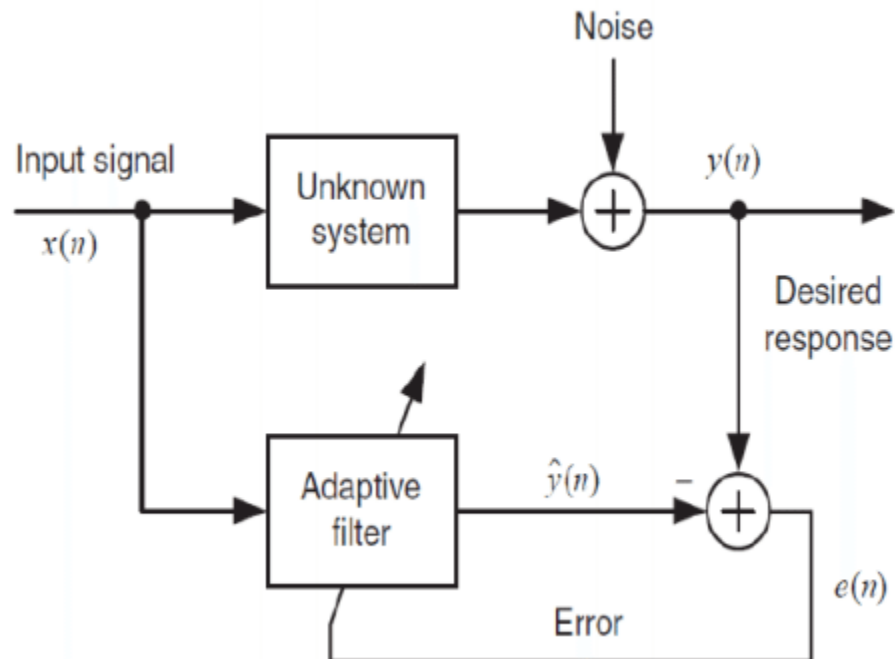


Figure 2.3: Block diagram for system identification application of adaptive filter. After [14].

Let the input signal at time n be denoted by the set of samples: $x(n), x(n - 1), \dots, x(n-M+1)$, where M is the number of adjustable parameters in the model.

The model and the plant both are applied with the same input signal. Let $d(n)$ and $y(n)$ represent the respective outputs of the plant and model. The adaptive filtering approach which is used to modify the model parameters uses the plant output $d(n)$ as a desirable response. The model's result is provided by

$$y(n) = \sum_{k=1}^M \hat{w}(n)_k x(n - k) \quad (2.2)$$

where $\hat{w}_1(n), \hat{w}_2(n), \dots$, and $\hat{w}_M(n)$ are the estimated model parameters. The output of a time-varying plant is non-stationary, and the desired response that the adaptive filtering algorithm is given is also non-stationary. In such a case, the adaptive filtering algorithm's job is to continuously monitor changes in the dynamics of the plant over time while still maintaining the modeling error low.

2.2 Adaptive filtering algorithms

Adaptive filters can be based on various basic algorithms as discussed here.

2.2.1 The Least Mean Square (LMS) Algorithm

In adaptive filtering, the LMS algorithm is by far the most popular algorithm. The low computational complexity, convergence in stationary environments, unbiased convergence in the mean, and stable behaviour when implemented with finite-precision arithmetic are the major characteristics that draw the use of the LMS algorithm.

The main advantage is the simplicity of the LMS method, which sets it apart from other linear adaptive algorithms. The cost function of the LMS algorithms is given as,

$$J(n) = |e(n)|^2 \quad (2.3)$$

where $|\cdot|$ represents the Euclidean norm and $e(n)$ gives the error signal, which is given by the difference between the desired signal and the filter output signal,

$$e(n) = d(n) - y(n) \quad (2.4)$$

$$e(n) = d(n) - \hat{w}^T(n) x(n) \quad (2.5)$$

where $\hat{w}^T(n)$ is the filter represented by a $M \times 1$ tap-weight vector and $x(n)$ is the $M \times 1$ input signal.

The gradient vector of $J(n)$ can be given as,

$$\partial J / \partial \hat{w} = -p + R \hat{w}(n) \quad (2.6)$$

R represents the correlation matrix of the received input signal and p is the cross-correlation vector between the received signal and the desired signal. The optimum solution of such a linear filter is given as,

$$\hat{w}_o = R^{-1}p \quad (2.7)$$

The filter coefficient vector can be updated as,

$$\hat{w}(n+1) = \hat{w}(n) - \mu \partial J / \partial \hat{w} \quad (2.8)$$

$$= \hat{w}(n) + \mu x(n) e(n) \quad (2.9)$$

where μ is the step-size parameter which controls the convergence and the steady state behavior of the LMS.

2.2.2 The Normalised Least Mean Square (NLMS) Algorithm

The NLMS algorithm, also known as normalised least mean square (NLMS), is a companion to the conventional LMS method. The NLMS algorithm can be thought of as a natural modification of the conventional LMS. Since the NLMS technique uses a variable convergence factor with the goal of minimizing the immediate output error, it typically converges more quickly than the LMS algorithm [16], [17].

Let $\hat{w}(n)$ represent the weight vector of the filter and $\hat{w}(n+1)$ represent the updated weight vector at next iteration. It can then be developed that the criterion for designing the normalized LMS algorithm as that of constrained optimization as:

The input vector $x(n)$ and the desired response $d(n)$, determine the updated weight vector $\hat{w}(n+1)$ so as to minimize the squared Euclidean norm of the deviation or change which is given as:

$$\delta \hat{w}(n+1) = \hat{w}(n+1) - \hat{w}(n) \quad (2.10)$$

The equation 2.10 represents the change in weight vector $\hat{w}(n+1)$ with respect to its previous value $\hat{w}(n)$, and is subjected to the constraint given as:

$$\hat{w}^T(n+1) x(n) = d(n) \quad (2.11)$$

The cost function can be given as:

$$J(n) = \|\hat{w}(n+1) - \hat{w}(n)\|^2 + \text{Re}\{\lambda^* [d(n) - \hat{w}^T(n+1)x(n)]\} \quad (2.12)$$

Here λ represents the Lagrange multiplier. The concept of Lagrange multipliers is used to obtain the solution of this optimization problem, which is given by the following filter coefficient update equation,

$$\hat{w}(n+1) = \hat{w}(n) + [\mu / \|x(n)\|^2] x(n) e(n) \quad (2.13)$$

It is clearly shown in the equation 2.13, that the reason why the term normalized is used. Therefore, the NLMS algorithm can also be stated as LMS algorithm with a time-variant step-size parameter. We can say that this is the reason for the NLMS algorithm exhibiting a faster convergence rate as compared to that shown by the conventional LMS algorithm.

Figures 2.4 and 2.5 show the comparison of convergence rate plots for LMS and NLMS algorithm when applied to a random signal. The convergence rate is plotted for different values of step-size parameter μ .

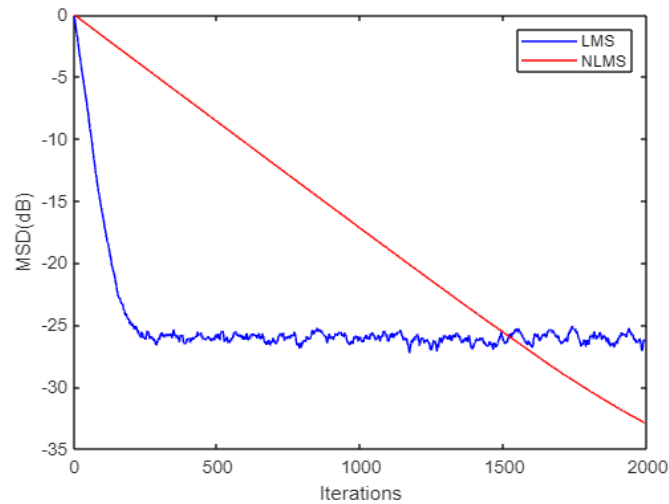


Figure 2.4: Convergence rate plot for LMS and NLMS for $\mu=0.02$

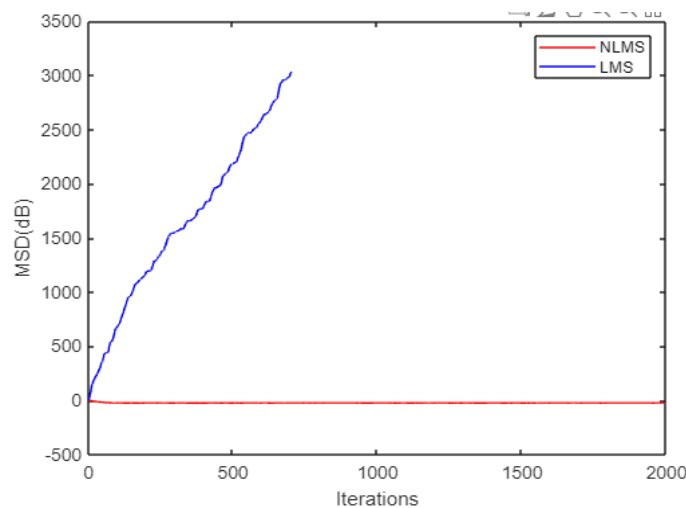


Figure 2.5: Convergence rate plot for LMS and NLMS for $\mu=0.5$

By observing the figure 2.4, it can be stated that LMS algorithm converges for smaller values of μ . Here, for $\mu=0.02$, LMS converges but NLMS doesn't converge, rather a straight line is obtained.

In figure 2.5, a large value of μ is taken. Here, for $\mu=0.5$, LMS diverges while NLMS algorithm converges. Normally, to obtain a better convergence rate, the range for μ is, $0 < \mu < 2$ for NLMS algorithm. Hence, it can be seen that the value of μ plays a vital role to obtain better convergence rates for respective algorithms. The significance of value of μ is explained further.

At each update step, the step-size (μ) determines how far we can advance along the error function surface. We can see that to prevent shifting the coefficient vector in a direction towards a higher squared error, μ must undoubtedly be selected such that $\mu > 0$. Furthermore, since the gradient of the cost function is computed using a local approximation of the autocorrelation matrix R in the LMS algorithm, the cost function at each time instant differs from a correct global cost function. Additionally, an excessively large step-size makes the LMS algorithm unstable, causing the coefficients to oscillate rather than converging to the fixed values and yielding the upper bound for μ . Therefore the choice of step-size parameter (μ), determines the convergence rate of the algorithm strictly.

2.2.3 The Proportionate Normalised LMS (PNLMS) Algorithm

When the ideal weight vector is sparse, the proportionate algorithm is one method from a class of algorithms that updates each filter coefficient proportionally to its magnitude, which speeds up convergence [19].

The available adaptation energy is distributed unevenly among the taps in the PNLMS algorithm as opposed to the NLMS method [1], [20]. The PNLMS algorithm works well in such cases.

Due to its extremely quick initial convergence rate, the PNLMS algorithm is advantageous for applications like network echo cancellation. However, after the initial phase, it significantly slows down and even outpace the NLMS algorithm in speed. The improved PNLMS (IPNLMS) algorithm and μ -law PNLMS (MPNLMS) algorithm [21] are the two modifications which have been developed to address this issue in order to maintain the rapid initial convergence speed throughout the entire adaptation process until the adaptive filter achieves its steady-state.

Let's say a sparse system is characterised by a L by 1 finite-impulse response filter coefficient vector w_o , where $x(n) = [x(n), x(n-1), \dots, x(n-L+1)]^T$ and $y_d(n) = w_o^T x(n) + v(n)$ are the input and output, respectively. $v(n)$ is an observation noise that is typically characterised as zero mean, Gaussian, independent identically distributed, and independent of $x(m)$ for any n, m . The vector w_o is determined by updating a filter coefficient vector $w(n) = [w_0(n), w_1(n), \dots, w_{L-1}(n)]^T$ as

$$w(n + 1) = w(n) + \mu G(n) x(n)e(n) / (x^T(n)G(n)x(n) + \delta_P) \quad (2.14)$$

where $G(n)$ is a diagonal gain matrix, with $g_i(n) \equiv [G(n)]_{i,i}$ satisfying $\sum_{i=0}^{L-1} g_i(n) = 1$ for $(0 < g_i(n) < 1)$, δ_P is a very small positive constant used to avoid division by zero, μ is the step size, and $e(n) = y_d(n) - w^T(n)x(n)$ is the filter output error.

Figure 2.6 and 2.7 shows the plot showing the comparison of convergence rates of NLMS and PNLMS for a sparse signal for two different values of step-size parameter, μ .

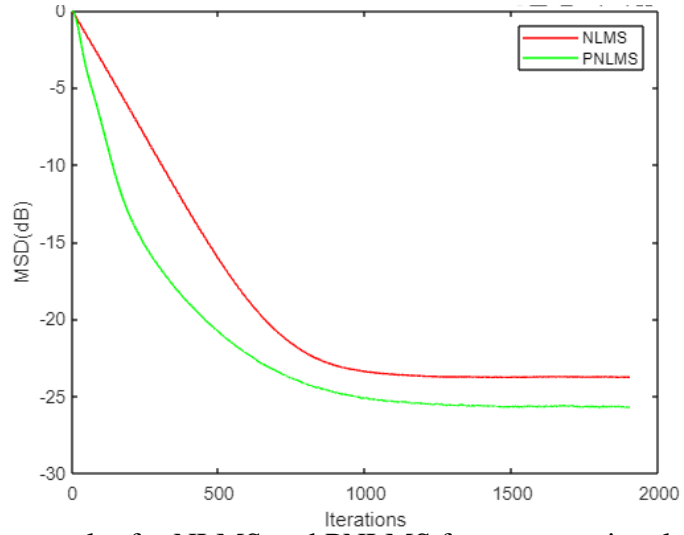


Figure 2.6: Convergence rate plot for NLMS and PNLMS for a sparse signal for $\mu=0.5$ (Instants in the plot represent the iterations)

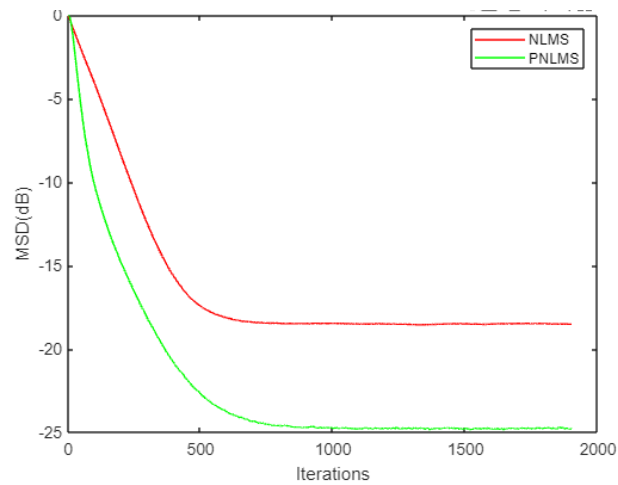


Figure 2.7: Convergence plot for NLMS and PNLMS for a sparse signal for $\mu=1$ (Instants in the plot represent the iterations)

From figures 2.6 and 2.7, it can be evidently understood that in case of a sparse signal or sparse systems, PNLMS algorithm much faster as compared to NLMS algorithm. Hence, we can say that PNLMS algorithm is exclusively used for a sparse entity.

Thus the proportionate normalised least-mean-squares (PNLMS) algorithm exploits the sparseness of impulse responses to achieve significantly faster adaptation than the conventional normalized least-mean-squares (NLMS) algorithm [1].

CHAPTER 3

Sparsity and Sparsity-aware algorithms

The issue of sparse system identification is briefly presented in this chapter.

3.1 Sparse-system identification

The impulse responses of unknown systems can typically be assumed to be sparse, with only a few significant coefficients scattered throughout a large number of inconsequential ones. The main goal of sparse system identification is to attempt to utilise sparse prior data to enhance the efficiency of filtering and estimation. A subset selection method was applied throughout the filtering process in various sparsity-exploiting algorithms in the past. This scheme was accomplished using statistical detection of active taps or sequential partial updating.

Other variations, like the proportionate normalised least-mean square (PNLMS) algorithm and its variants [1], [19], [20], [23], assign proportional step sizes to various taps based on their magnitudes [24].

The desired signal in a sparse system identification application, is produced when an unknown sparse system is activated by an input signal. Additionally, the input signal is used as an input for an adaptive filter $w(n)$ with M coefficients, which generates an output estimate $y(n)$, which is then contrasted with the reference signal $d(n)$. The error signal $e(n)$ is made up of the difference between the output of the sparse adaptive filter y and the desired signal $d(n)$. The adaptive filter represents a model for the unknown sparse system when the output error $e(n)$ is minimised.

Fig. 3.1 displays the sparse system identification block diagram. Here, the unknown sparse system is subjected to an input signal $u(n)$, with N samples, and $d(n)$ is the reference signal, which is also the response signal. The issue that needs to be resolved is how to use an adaptive algorithm to recognise and make use of the system's sparseness in order to identify the unknown sparse system.

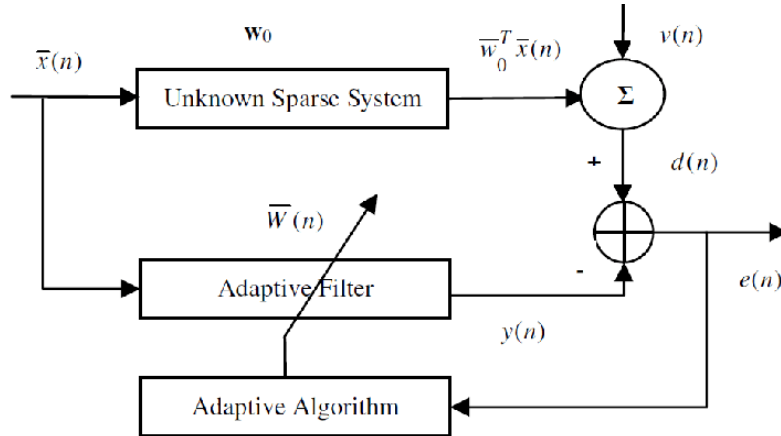


Figure 3.1: Block diagram of sparse-system identification using an adaptive algorithm.

There are numerous distinct kinds of sparse systems. In a general sparse system, the non-zero coefficients can be placed randomly. In the meantime, there is a distinct family of systems called as block-sparse systems or clustering-sparse systems [4]. One typical characteristic of the network echo path is a bulk delay that depends on network loading, encoding, and jitter buffer delays. The impulse response is dominated by ‘inactive’ areas with coefficient magnitudes that are near to zero [1], which produce a ‘active’ region with a duration of between 8 and 12 ms. A typical single clustering sparse system is the network echo response.

3.2 PNLMS Algorithm

Due to its extremely quick initial convergence rate, the PNLMS algorithm is advantageous for applications like, sparse-system identification, for example in network echo cancellation. However, after the initial phase, it significantly slows down and even outpace the NLMS algorithm in speed. The improved PNLMS (IPNLMS) algorithm and μ -law PNLMS (MPNLMS) algorithm [20] are the two modifications which have been developed to address this issue in order to maintain the rapid initial convergence speed throughout the entire adaptation process until the adaptive filter achieves its steady-state.

The objective of the PNLMS algorithm is to minimize the cost function. The weight update equation is given as:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \mathbf{G}(n) \mathbf{x}(n)e(n) / (\mathbf{x}^T(n)\mathbf{G}(n)\mathbf{x}(n) + \delta_p) \quad (3.1)$$

<p>Initialization:</p> $\hat{\mathbf{w}}(0) = [0, 0, \dots, 0]^T$
<p>Update for each time instant: $n > 0$</p> $l_\infty(n+1) = \rho \max\{\delta, \ \hat{w}_1(n)\ , \ \hat{w}_2(n)\ , \dots, \ \hat{w}_M(n)\ \}$ $l_k(n+1) = \max\{l_\infty(n+1), \ \hat{w}_k(n)\ \}$ $g_i(n+1) = \frac{l_k(n+1)}{\frac{1}{M} \sum_{i=1}^M l_i(n+1)} \quad 1 \leq i \leq M$ $G(n+1) = \text{diag}\{g_1(n+1), g_2(n+1), \dots, g_M(n+1)\}$ $\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) + \frac{\mu G(n+1) \mathbf{u}(n) e^*(n)}{\mathbf{u}^T(n) G(n+1) \mathbf{u}(n)}$ $n = n + 1$

Table 3.1: Summary of PNLMS algorithm

In the equations given in the table 3.1, M is the length of the adaptive filter, μ is the step-size parameter, δ is a small positive number used to avoid overflowing, $\hat{w}_k(n)$ is the k th coefficient of $\hat{\mathbf{w}}$ at time n . The constant δ is important when all the coefficients are zero initially and, together with ρ , prevent the very small coefficients from stalling. The notation l_∞ is intended to reflect an interpretation as the norm of the vector $\hat{\mathbf{w}}_k(n)$. The new parameters, ρ and δ reflect small-signal regularization.

3.3 Zero-Attracting NLMS (ZA-NLMS) Algorithm

The l_1 -norm optimization method is used with the traditional NLMS algorithm to harness the sparseness. In this algorithm, the NLMS cost function is conjuncted with the l_1 -norm penalty on the coefficients. The goal of the ZA-NLMS is to minimize the cost function given as:

$$\mathbf{J}(n) = \|\hat{\mathbf{w}}(n+1) - \hat{\mathbf{w}}(n)\|^2 + \|\hat{\mathbf{w}}(n)\|_1 \quad (3.2)$$

The equation (3.2) is subjected to constraint given in equation (3.3).

$$\mathbf{d}(n) - \hat{\mathbf{w}}^T(n+1)\mathbf{u}(n) = 0 \quad (3.3)$$

Further we use the Lagrange multiplier method to solve this constrained optimization problem. Then the cost function can be given as:

$$J(n) = \|\hat{w}(n+1) - \hat{w}(n)\|^2 + \text{Re}\{\lambda^* [d(n) - \hat{w}^T(n+1)u(n)]\} + \|\hat{w}(n+1)\|_1 \quad (3.4)$$

The partial derivative of $J(n)$ with respect to $\hat{w}(n+1)$ can be derived as:

$$\delta J(n) / \delta \hat{w}(n+1) = \hat{w}(n+1) - \hat{w}(n) - u(n)\lambda + \alpha \text{sgn}[\hat{w}(n+1)] \quad (3.5)$$

where $\text{sgn}[\cdot]$ gives the sign of the arguments.

By solving the above equations, we can get the coefficient update equation as:

$$\hat{w}(n+1) = \hat{w}(n) + (\mu/\|u(n)\|^2)u(n)e(n) + \alpha(u(n)u^T(n)/\|u(n)\|^2)\text{sgn}[\hat{w}(n)] - \alpha \text{sgn}[\hat{w}(n)] \quad (3.6)$$

By contrasting the traditional NLMS function with the ZA-NLMS update equation, we can observe an additional term called $\alpha \text{sgn}[\hat{w}(n)]$, which pulls the tap coefficients toward zero. This is called the zero attractor feature, and α determines its strength. It makes sense that when the bulk of the coefficients of \hat{w} are zero, or when the system is sparse, the zero attractor will speed-up the convergence.

3.4 Zero-Attracting PNLMS (ZA-PNLMS) Algorithm

After around a couple of hundreds of iterations, the PNLMS algorithm's convergence starts to slow down. This suggests that in the PNLMS method, the active taps converge quickly, in approximately 250 iterations, but the inactive taps takes longer time to converge [9]. This slow down of convergence takes place because the effective step sizes of inactive taps gradually decrease. However, in the ZA-PNLMS algorithm, the inactive taps are also subjected to a force from the zero attractors, which work to nudge them towards their true value, which is zero. As a result, even after 250 iterations, the ZA-PNLMS algorithm can still maintain a large portion of their original fast convergence rates.

In addition, the ZA-PNLMS algorithm maintains a lower steady state EMSE as compared to the PNLMS algorithm. However, this only occurs when the active taps

have sufficiently large magnitudes because, in these circumstances, the effect of the zero attractors on the active taps after their convergence is negligible. As a result, one obtains similar performance whether the zero attractors' action is present or suppressed.

The coefficient or weight update equation for ZA-PNLMS algorithm is given as:

$$\hat{w}(n+1) = \hat{w}(n) + \{\mu e(n)G(n)u(n)/(u^T(n)G(n)x(n) + \delta_p)\} - \rho \text{sgn}(w(n)) \quad (3.7)$$

The equation (3.7) gives the ZA-PNLMS algorithm, the second term on the right side of the equation is the PNLMS update term and the last term is the zero attractor term.

These algorithm were simulated for identifying a sparse system with impulse response of a sparse system shown in figure 3.2. The system has a total of 100 taps or impulse response coefficients, of which only 10 are active, i.e., non-zero and the system input was taken to be a zero mean, unit variance white random process.

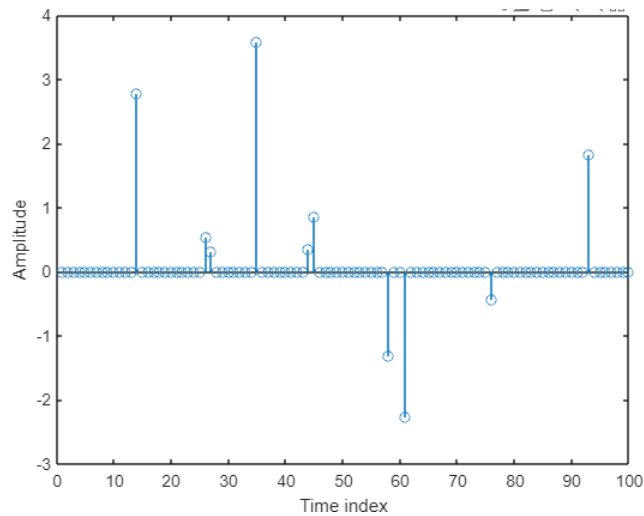


Figure 3.2: Impulse response of a sparse system.

The figures 3.3 and 3.4 shows the comparison of convergence plots for the PNLMS, ZA-NLMS and ZA-PNLMS algorithms for different values of step-size parameter, μ .

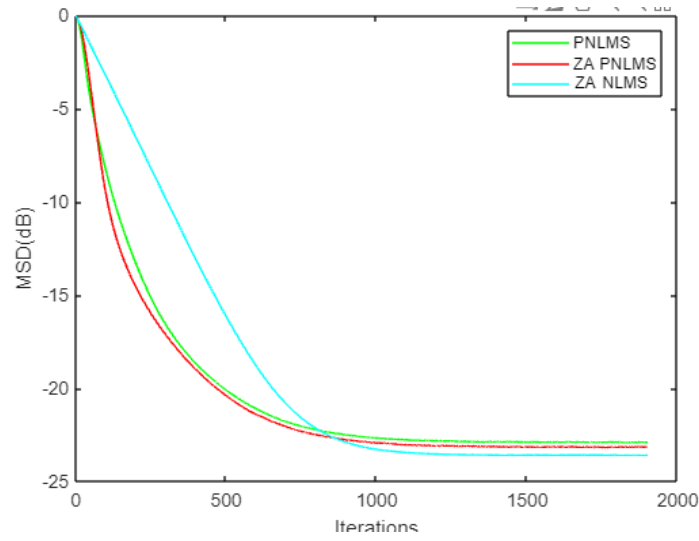


Figure 3.3: Convergence plots for PNLMS, ZA-NLMS AND ZA-PNLMS for $\mu=0.5$

It is clearly evident from figure 3.3 that in case of sparse-system identification, PNLMS and ZA-PNLMS perform better than ZA-NLMS. The convergence in case of PNLMS and ZA-PNLMS is identical initially. Next, in the figure 3.4 the performance of these three algorithms for value of μ equals to 1, is shown.

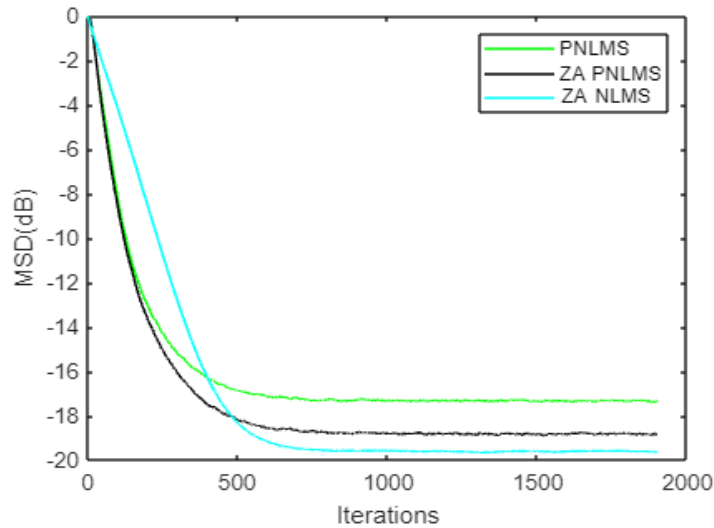


Figure 3.4: Convergence plots for PNLMS, ZA-NLMS AND ZA-PNLMS for $\mu=1$

The convergence plots in figure 3.4 are obtained for $\mu=1$, as $\mu=1$ gives a balance between the steady-state performance and the convergence rate for PNLMS, ZA-NLMS as well as for ZA-PNLMS.

It can be noted from above plots that the PNLMS and ZA-PNLMS converge identically initially and more quickly than ZA-NLMS for a sparse system. It is also noted that the steady state results are better or compensated in case of ZA-NLMS for a sparse system. However, ZA-PNLMS gives better convergence and steady state result as compared to PNLMS because of the zero-attractor feature which helps to maintain the convergence rate even for small or inactive taps.

CHAPTER 4

Block-Sparsity and Proposed algorithm

Numerous applications, including acoustic echo cancellation (AEC), industrial interference suppression, and biomedical, usually include system identification. Iteratively identifying the unknown system often uses the LMS algorithm or the normalised form to adjust to the time-varying properties of the statistical speech signal. There are few non-zero or high magnitude coefficients and numerous tap-weights with zero or tiny values in various cases, such as network echo cancellation, where the impulse responses are sparse. There are numerous types of sparse systems. A block-sparse system or a block-compressible system is a typical one, such as TV broadcast channels [25]. Block-sparse systems' nonzero coefficients are made up of one or more clusters, which are collections of big or non-zero coefficients. This is in contrast to general impulse response sparse systems, in which large coefficients are scattered at random.

A recent block-sparse system proposal, known as the BS-PNLMS [5], proposes the use of mixed $l_{2,1}$ norm rather than l_1 norm for gain computation. The l_2 norm of each equi-partitioned block replaces the gains, which are assigned proportionally to the tap-weights in PNLMS. This algorithm performs poorly in steady-state conditions while having a very quick initial convergence. Furthermore, for optimal performance, the cluster and block sites must exactly overlap. This algorithm's performance is highly dependent on the cluster placement. Practically speaking, it is challenging to make such an assumption. Additionally, a priori information of the original system's active cluster size is assumed to be present, which may not always be the case.

Further, another block-sparse system identification algorithm is proposed which is more effective than the current sparsity-induced block-sparse system identification techniques in terms of convergence speed [26]. The suggested approach is based on a proportionate gain control inspired by a $l_{2,1}$ norm and a regularisation penalty with a $l_{2,0}$ norm for quick convergence compared to existing algorithms.

4.1 Improved Block-Sparse PNLMS (IBS-PNLMS) Algorithm

The IBS-PNLMS method, which is a descendant of the PNLMS algorithm, accelerates the convergence rate of its parent algorithm by computing block gains rather than determining the gains of each individual coefficient.

Let the unknown system taps or the coefficients are given as $u = [u_0 u_1 \cdots u_{L-1}]^T$ and the input to the system is given as $x = [x_n x_{n-1} \cdots x_{n-L+1}]^T$, and the length of the unknown system is given by L , and u is known a-priori to be block-sparse.

Here, the real tap-weights are considered, and $[\cdot]^T$ denotes the transpose operation. The intended response for the sparsity-induced adaptive filter may then be described as the noisy output from this system and is given by:

$$\mathbf{d}_n = \mathbf{x}_n^T \mathbf{u} + v_n \quad (4.1)$$

\mathbf{d}_n is the noisy system output and v_n is the additive noise, and it is considered to be mutually independent of the filter tap-weights \mathbf{u} . So, the error, which is the difference between the desired signal and the estimated adaptive filter's output, can be written as follows:

$$\mathbf{e}_n = \mathbf{d}_n - \mathbf{x}_n^T \mathbf{w}_n \quad (4.2)$$

where $\mathbf{w}_n = [w_{0n} \ w_{1n} \ \cdots \ w_{L-1n}]^T$ denotes the estimated filter tap weights at time index n .

The convex minimization problem is formulated by using the $l_{2,1}$ norm of \mathbf{w} , and that is given as follows:

$$\|\mathbf{w}\|_{2,1} \triangleq \left\| \begin{bmatrix} \|\mathbf{w}_{[1]}\|_2 \\ \|\mathbf{w}_{[2]}\|_2 \\ \vdots \\ \|\mathbf{w}_{[N]}\|_2 \end{bmatrix} \right\|_1 = \sum_{i=1}^N \|\mathbf{w}_{[i]}\|_2$$

We can define, $\mathbf{w}_{[k]} = [w_{(k-1)P+1}, w_{(k-1)P+2}, \dots, w_{kP}]$ as the k th group of the coefficient vector \mathbf{w} , in a manner that the total number of groups are N and each group has P number of coefficients. The above mixed norm is minimized in order to solve a convex optimization problem, which is given as:

$$\min \|\mathbf{w}\|_{2,1}$$

Now to derive the weight update equation for IBS-PNLMS algorithm, the following cost function is minimized,

$$\mathbf{J}(\mathbf{n}) = \|\mathbf{w}_{n+1} - \mathbf{w}_n\|_G^{-1} + \zeta \|\mathbf{w}_n\|_{2,0} \quad (4.3)$$

Here, $\|\mathbf{x}\|_A^{-1}$ denotes a generalized inner-product $\mathbf{x}^T \mathbf{A} \mathbf{x}$.

The above cost function is subjected to the constraint given as:

$$\mathbf{d}_n = \mathbf{x}_n^T \mathbf{w}_{n+1} \quad (4.4)$$

Here ζ is a positive constant which regulates the zero point attraction term's contribution in a group. Then a Lagrange multiplier λ is used to solve the constrained cost function, given as:

$$\Psi_n = \|\mathbf{w}_{n+1} - \mathbf{w}_n\|_{G^{-1}} + \zeta \|\mathbf{w}_n\|_{2,0} + \lambda (\mathbf{d}_n - \mathbf{x}_n^T \mathbf{w}_{n+1}) \quad (4.5)$$

Now, differentiating Ψ_n with respect to \mathbf{w}_{n+1} and putting it equal to zero, we get

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \lambda \mathbf{G}_n \mathbf{x}_n + \zeta \mathbf{f}(\mathbf{w}_{n+1}) \quad (4.6)$$

And here, $\mathbf{f}(\mathbf{w}_n) = [f_1(\mathbf{w}_n), f_2(\mathbf{w}_n), \dots, f_N(\mathbf{w}_n)]$.

The detailed algorithm is given in table 4.1.

<p>Input: $\{x_n, d_n\}_{n=0,1,2,\dots}, L, N, P, \mu, \alpha, \epsilon, \delta, \kappa$ Output: $\{\mathbf{w}_n\}_{n=0,1,2,\dots}$ Initialization $\mathbf{w}_0 = \mathbf{0}, N = L/P, \rho = \mu\zeta$ for $n = 0, 1, 2, \dots$ do $e_n = d_n - \mathbf{x}_n^T \mathbf{w}_n$ for $i = 1, 2, \dots, N$ do $q_i = 1 - \exp(-\alpha \ \mathbf{w}_{[i]}\ _2)$ end for for $i = 1, 2, \dots, N$ do $V_i = \left(\sum_{(l=i-1)P+1}^{iP} w_{l,n} ^2 \right)^{1/2}$ $\theta_i = \max\{\delta \max\{\kappa, q_1 \dots q_N\}\}$ end for for $i = 1, 2, \dots, L$ do $g_i(n) = \frac{\theta_{\lfloor i/P \rfloor}}{N \sum_{m=1}^N \theta_m}$ $f_i(\mathbf{w}_n) = \alpha^2 w_{i,n} - \alpha w_{i,n} \max\left(\frac{1}{V_{\lfloor i/P \rfloor} + \epsilon}, \alpha\right)$ end for $\mathbf{G}_n = \text{diag}[g_1(n), g_2(n), \dots, g_L(n)]$ $\mathbf{f}(\mathbf{w}_n) = [f_1(\mathbf{w}_n), f_2(\mathbf{w}_n), \dots, f_L(\mathbf{w}_n)]$ $\mathbf{w}_{n+1} = \mathbf{w}_n + \mu \frac{e_n \mathbf{G}_n \mathbf{x}_n}{\mathbf{x}_n^T \mathbf{G}_n \mathbf{x}_n + \epsilon} + \rho \mathbf{f}(\mathbf{w}_n)$ end for</p>

Table 4.1: Improved BS-PNLMS Algorithm

The weight update equation for IBS-PNLMS algorithm is given as:

$$w_{n+1} = w_n + (\mu e_n G_n x_n / x_n^T G_n x_n) + \rho f(w_n) \quad (4.7)$$

Here, $\lceil \cdot \rceil$ is the ceiling function, α is a positive constant which is tuned as per the magnitude of the filter coefficients and $\rho = \mu \zeta$. Further, δ is introduced in the denominator to avoid division by zero.

4.2 Simulation result

In this section, simulation results are obtained to show the performance of the proposed sparsity-aware algorithm, IBS-PNLMS.

For the simulations, a white Gaussian random signal with a zero mean and unit variance, has been used as the input for both the unknown system and the adaptive filter. The length of the unknown system and the length of the adaptive filters has been taken as $L=120$ taps. The signal consists of 6 equi-partitioned tap blocks, and the block size is 20, and the active and inactive or non-zero taps are equal in each block.

The impulse response of the block-sparse signal is shown in figure 4.1.

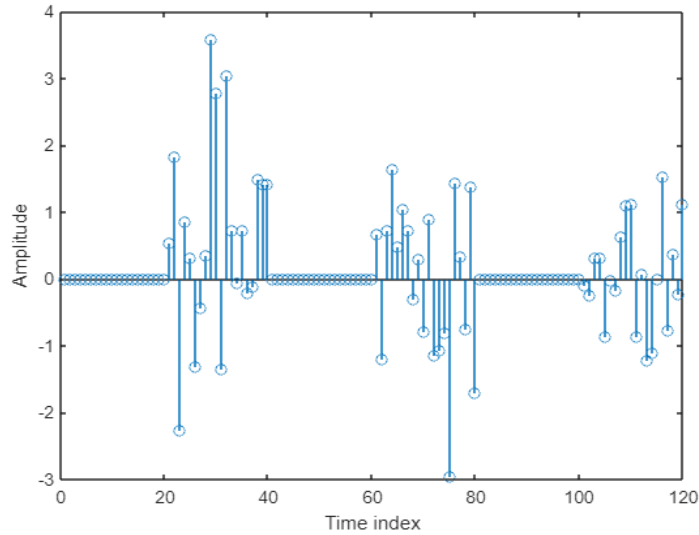


Figure 4.1: Impulse response of block-sparse system taken.

The performance of the IBS-PNLMS in terms of convergence is compared with the convergence in case of PNLMS and ZA-PNLMS for different values of step-size parameter, μ . The value of α is taken as 1 and $\delta = 0.01$, $\kappa = 0.01$, and $\epsilon = 10^{-16}$. The regularization parameter, $\rho = 0.00001$.

The figures 4.2 and 4.3 show the comparison of IBS-PNLMS convergence with the convergence of PNLMS and ZA-PNLMS algorithms for a block-sparse system, for different values of step-size parameter, μ .

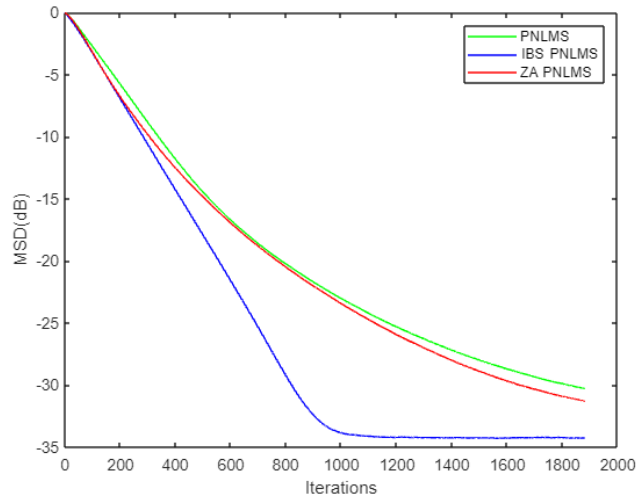


Figure 4.2: Convergence plots for PNLMS, ZA-PNLMS and IBS-PNLMS for $\mu=0.5$

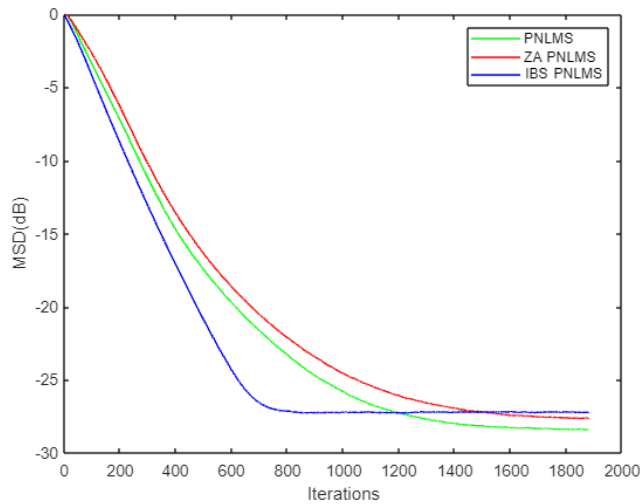


Figure 4.3: Convergence plots for PNLMS, ZA-PNLMS and IBS-PNLMS for $\mu=1$

It is evident from figures 4.2 and 4.3, that in case of block-sparse systems, IBS-PNLMS gives better convergence of tap weights or coefficients as compared to already established algorithms to harness sparsity.

It is also observed that for high value of the step-size parameter, μ , the error or mean standard deviation increases. Therefore, the accuracy of the method is compromised.

CHAPTER 5

Problem Statement and solution

In the previous section we discussed about the block-sparsity and proposed an algorithm, IBS-PNLMS for block-sparse system identification. It was made evident that the IBS-PNLMS algorithm performs better in terms of convergence for a block-sparse system as compared to the already existing algorithms used earlier to deal with sparse systems.

Now, in this section, we have tried to modify the function of weights incorporated in the weight update equation of IBS-PNLMS algorithm, which is given as:

$$w_{n+1} = w_n + (\mu e_n G_n x_n / x_n^T G_n x_n) + \rho f(w_n) \quad (5.1)$$

Here,

$$f_i(w_n) = \alpha^2 w_{i,n} - \alpha w_{i,n} \max \{ (1 / (V_{\Gamma(i/P)\Gamma} + \epsilon)), \alpha \}, \text{ for } 1 < i < L \quad (5.2)$$

In this section, the function, $f(w_n)$ is modified to enhance the convergence rate for a block-sparse system identification application.

Another recent proposed algorithm, μ -law PNLMS (MPNLMS), has been considered for the modification purpose [20]. The optimal convergence outcomes attained for the previous algorithms with various but fixed step-size control factors served as the inspiration for the MPNLMS algorithm. On the basis of this, we anticipate that the proposed algorithm will behave similarly following the initial phase, that is, when the step-size control variables do not vary much. This motivates the idea that the proposed approach will converge more quickly than the proposed IBS-PNLMS.

Here the function, $f(w_n)$ is defines as:

$$f_i(w_n) = (\ln(1 + \mu w_{i,n}) / \ln(1 + \mu)), \text{ for } 1 < i < L \quad (5.3)$$

The μ -law function mentioned above is only defined in the range, $[0,1]$. The MPNLMS method will fail if some of the active coefficient magnitudes fall outside of this range or if all of the active coefficient magnitudes are either too little or too huge.

The other parameters are kept same as taken in previous section for IBS-PNLMS implementation. The values of α are taken in the range $(-1 \leq \alpha \leq 1)$ and $\delta = 0.01$, $\kappa = 0.01$, and $\epsilon = 10^{-16}$. The regularization parameter, $\rho = 0.00001$.

The figures 5.1 and 5.2 show the simulation results to obtain convergence plot for the proposed modified IBS-PNLMS for different values of step-size parameter, μ .

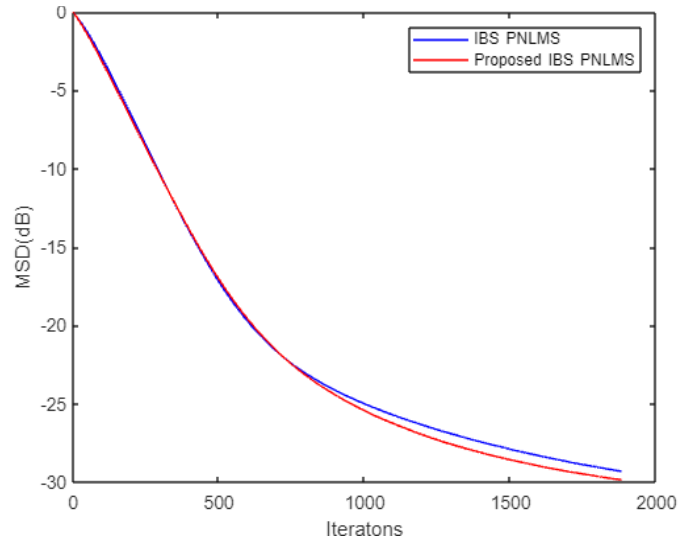


Figure 5.1: Convergence plots for IBS-PNLMS and the modified IBS-PNLMS, for $\mu = 0.5$

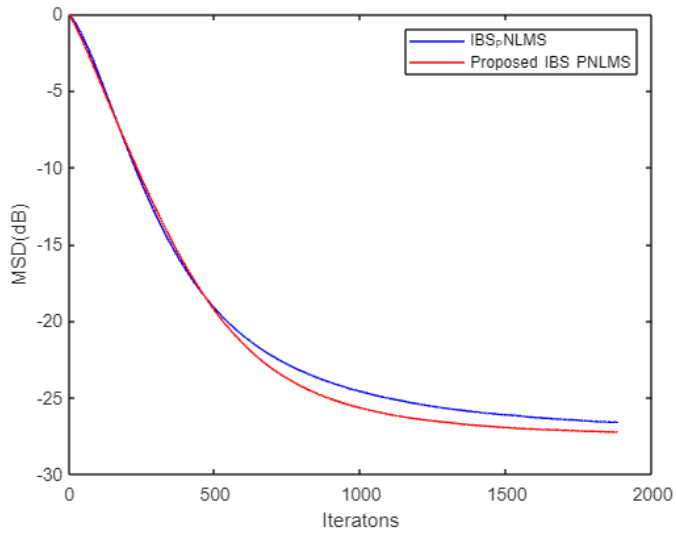


Figure 5.2: Convergence plots for IBS-PNLMS and the modified IBS-PNLMS, for $\mu = 1$

The figures 5.1 and 5.2 show the comparison of convergence plots for IBS-PNLMS and the modified IBS-PNLMS algorithms. It is noted that in case of the modified IBS-PNLMS, the initial convergence is identical to the convergence rate of existing IBS-PNLMS. However, for the modified IBS-PNLMS, the steady state response is better as compared to the original algorithm.

The identical initial convergence gives us the motivation to explore this method more and to do more modification in order to derive better convergence results. This motivation lays the foundation to implement μ -law PNLMS and its modifications for the application of system identification in case of block-sparse systems for future work.

CHAPTER 6

Conclusions and future scope

6.1 Conclusions

In this thesis, the sparsity, and then block-sparsity, of signals and systems has been harnessed and studied and some low-complexity adaptive filtering algorithms have been derived and studied.

Chapter 2 provides a summary of adaptive filtering and its application. The mostly used application of adaptive filters, system identification has been discussed in detail. Also, some commonly used algorithms for system identification, like, LMS, NLMS and PNLMS are discussed. We were able to conclude and compare the performance of these algorithms in system identification application. It is observed that for different values of step-size parameter, μ , how LMS and NLMS convergence vary. It is noted that for very low values of μ , LMS gives better convergence of system coefficients as compared to NLMS algorithm. Also, on the other hand, for high values of μ , NLMS performs better than LMS. Then with the help of experiment result in form of the convergence plot, it could be established that in case of sparse signal or system identification, PNLMS gives better convergence as compared to NLMS algorithm.

Then the sparsity is introduced and the sparse system identification is discussed. The zero-attractor feature of the Zero-Attracting variant of NLMS and PNLMS algorithms is introduced. The additional zero-attractor term helps to pull the tap coefficients towards zero. It also helps to maintain a lower steady state EMSE as compared to the PNLMS algorithm. It was shown with the help of convergence plots for PNLMS and ZA-PNLMS, that how the zero attracting variant produces more optimized convergence results as compared to the parent algorithm, PNLMS.

Further, moving ahead, block-sparsity is introduced and IBS-PNLMS algorithm is proposed. The convergence rate of PNLMS is compared with that of IBS-PNLMS algorithm for an equi-partitioned block-sparse system, with block-size, $P=20$. It is observed that IBS-PNLMS algorithm gives better convergence of coefficients for a block-sparse system than the PNLMS and ZA-PNLMS algorithms.

Then the IBS-PNLMS algorithm is modified to obtain better convergence rate. However, it was noticed that the initial convergence came out to be same as that of the original IBS-PNLMS with better steady state response.

6.2 Future Scope

In this thesis, identification of sparse signals has been implemented using already existing algorithms as well as with modified IBS-PNLMS. The order of the filter and the system is taken to be less.

Therefore, one of the possible future works is to investigate higher order systems. Also, in the experiments the general sparse signals are taken, in future work, more practical signals could be taken to identify sparse signals or systems.

In chapter 5, it was observed that in case of modified IBS-PNLMS the identical initial convergence gives us the motivation to explore this method more and to do more modification in order to derive better results in terms of convergence rate and mean standard deviation as well. This motivation lays the foundation to implement μ -law PNLMS (MPNLMS) and its modifications for the application of system identification in case of block-sparse systems for more practical applications for the future work.

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