Adaptive Channel Estimation and Loading for OFDM Based Two-Way Relay Systems

by

Arun Joy

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Declaration

This is to certify that

1. the thesis comprises my original work towards the degree of Doctor of Philosophy in Information and Communication Technology at DA-IICT and has not been submitted elsewhere for a degree,
2. due acknowledgment has been made in the text to all other material used.

Arun Joy

Certificate

This is to certify that the thesis work entitled "Adaptive Channel Estimation and Loading for OFDM Based Two-Way Relay Systems" has been carried out by Arun Joy (200821002) for the degree of Doctor of Philosophy in Information and Communication Technology at this Institute under my supervision.

C. Vijay Kum 6/3/2014
Dr. Vijay Kumar Chakka

Abstract

In this thesis we introduce a low complexity Recursive Least Square (RLS) based Orthogonal Frequency Division Multiplexing (OFDM) channel estimation for two-way relay system. Using the estimate of the channel, a bit and power loading scheme for OFDM based two-way relay system is implemented.

In a wireless communication scenario, the channel frequency response is usually correlated across both time and frequency. Hence an optimal Minimum Mean Square Error (MMSE) estimator for OFDM channel is one which considers the correlation in the dimension of time and frequency. These estimators are known as 2D estimators. Since the channel is time varying and the channel statistics are not available, we implement a 2D-RLS adaptive filter based channel estimator.

The 2D-RLS filter used for channel estimation is based on the principle of fast array based algorithms. The advantage of this algorithm is that it has a computational complexity comparable to that of 2D-Normalized Least Mean Square (2D-NLMS) algorithm while having same convergence rate as the conventional 2D-RLS algorithm. The reason for the low complexity of fast array algorithm is due to the fact that it considers the time shift structure in the input data vector while updating the inverse of covariance matrix. In the case of OFDM channel, this time shift structure is not directly evident. Hence we propose a simple reordering of the input data vector so as to bring in the notion of time shift structure. The adaptive filter so proposed is called Fast Array Multichannel 2D-RLS (FAM 2D-RLS) filter. In order to reduce the number of training symbols, the OFDM adaptive channel estimator is implemented based on the principle of Decision Directed Channel Estimation (DDCE).

The standard literature on adaptive filter usually assumes that the data to be estimated is a scalar quantity while the weight is a vector. But in our proposed OFDM channel estimation method, the data to be estimated is a vector and the weight is a matrix. The steady state analysis of the RLS filter with weight matrix is derived and verified using simulations. The proposed steady state analysis is derived based on the fact that any adaptive filter can be viewed as an iterative equation solver, with RLS algorithm being a special case. Hence this method could be used for deriving the steady state analysis of any adaptive filter algorithm which has a weight matrix.

A two-way relaying scheme is a spectrally efficient relaying scheme. Using FAM 2D-RLS, an OFDM-DDCE is proposed for this relaying scheme. It is observed that even a simple case of relaying involving nodes and relay with single antenna requires the concepts of Multiple Input Multiple Output (MIMO) systems for estimating the channel. The performance of the FAM 2D-RLS for estimating pedA, pedB and vehA channel in case of two-way relay are analyzed. The

complexity of FAM 2D-RLS can be further reduced, if instead of considering the correlation of the frequency response across all the frequency samples, we consider only a block of the frequency response, i.e the channel frequency response vector is grouped into M subvectors and is estimated using M parallel FAM 2D-RLS filters. This estimation algorithm is called Block FAM 2D-RLS (BFAM 2D-RLS). The computational complexity of BFAM 2D-RLS is lesser compared to that of FAM 2D-RLS by a factor of $\frac{1}{M}$. It is observed that even though BFAM 2D-RLS does not consider the correlation across all the frequency response samples, the Mean Square Error (MSE) of the estimate is comparable to that of FAM 2D-RLS. It is also observed that MSE of channel estimation for BFAM 2D-RLS with large M is lesser compared to FAM 2D-RLS for the case of highly frequency selective channels like pedB and vehA.

The capacity and Bit Error Rate (BER) of a communication system can be improved if the modulation scheme and transmitted power is adapted with respect to the variation of the channel. In order to perform adaptive modulation, the channel State Information (CSI) should be available at the transmitting node. In the case of Adaptive OFDM (AOFDM) systems, based on Signal to Noise Ratio (SNR) of each subchannel, power and bit could be assigned. This is known as power and bit loading algorithms. In a two-way relay system, if the channel estimation is performed at the node, the channel response so obtained pertains to that of the overall channel between the two transmitting nodes. In order to implement loading algorithms, the nodes require the CSI of individual channel, i.e channel between source - relay and relay - destination. Since our proposed channel estimation method is implemented in the frequency domain, the individual channel can be easily obtained from the estimated combined channel with a sign ambiguity. The effect of channel estimation error on loading algorithms for two-way relay is also analyzed. All the computer simulations are performed using MATLAB®.

As a summary of this section, we point out the unique features of this thesis,

1. A low complexity adaptive filter called FAM 2D-RLS filter is proposed for DDCE-OFDM.
2. Steady state equations for RLS filter with matrix weight is derived.
3. FAM 2D-RLS based DDCE-OFDM is implemented for two-way relay systems.
4. Complexity of FAM 2D-RLS is further reduced by BFAM 2D-RLS.
5. A loading algorithm is implemented for OFDM based two-way relay systems.
6. Effect of channel estimation error on loading algorithm for two-way relay system is analyzed

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Contents

1 Introduction **1**

1.1 Publications 3

1.1.1 Journal 3

1.1.2 Conference 3

1.2 Motivation 3

1.2.1 System Model 4

1.2.2 Linear Transceiver Filter 5

1.2.3 Simulation Results 7

1.2.4 Open Problems 10

1.3 Thesis contribution 11

2 Fast Array Multichannel 2D-RLS Based OFDM Channel Estimator **12**

2.1 Introduction 12

2.2 2D OFDM Channel Estimation 13

2.3 Fast Array Multichannel 2D-RLS 15

2.3.1 Adaptive Filter as an Iterative Equation Solver 16

2.3.2 Derivation of the FAM 2D-RLS 19

2.4 Steady State Analysis of FAM 2D-RLS 24

2.5 Simulation Setup 28

2.6 Results and Analysis 28

2.7 Conclusion 29

3 FAM 2D-RLS Based Channel Estimation for Two-Way Relay Systems **30**

3.1 Introduction 30

3.2 Two-Way Relay With Node Capability 31

3.2.1 System Model 31

3.2.2 Least Square Channel Estimation 33

3.2.3 DDCE-OFDM 34

3.3 Two-Way Relay with Relay Capability 35

3.3.1 System Model . . . 35
3.3.2 Least Square Channel Estimation . . . 37
3.3.3 DDCE-OFDM . . . 37
3.3.4 Transceiver Matrix . . . 38
3.4 BFAM 2D-RLS Based Channel Estimation . . . 38
3.5 Simulation setup . . . 41
3.6 Results and Analysis . . . 44
3.7 Conclusion . . . 50

4 Adaptive OFDM Based Two-Way Relay Systems **51**
4.1 Introduction . . . 51
4.2 System Model . . . 51
4.3 Obtaining Individual CSI from Overall CSI . . . 53
4.4 Adaptive OFDM for Two-Way Relay Systems . . . 53
4.4.1 Levin-Campello Loading Algorithm . . . 54
4.5 Channel Estimation Error and AOFDM . . . 56
4.6 Simulation setup . . . 57
4.7 Results and Analysis . . . 57
4.8 Conclusion . . . 60

5 Conclusion **65**

Appendix **66**

A OFDM **66**

B Adaptive Filter **68**
B.1 NLMS Algorithm . . . 69
B.2 RLS Algorithm . . . 69

C Acronyms **71**

D Notations **72**

List of Figures

1.1 Two-Way relay with relay capability 4
1.2 MSE performance of NLMS and LMS receive filter 8
1.3 MSE performance of NLMS and LMS transmit filter 9
1.4 MSE performance of PRA and NLMS receive filter 9
1.5 MSE performance of PRA and NLMS transmit filter 10

2.1 Adaptive DD channel estimator for OFDM system 15
2.2 2D OFDM Adaptive Channel Estimator as a Multichannel Adaptive Filter . . . 16
2.3 Theoretical and simulated MSE for FAM 2D-RLS 27
2.4 Learning curve for FAM 2D-RLS, 2D-NLMS & 2D-RLS 27
2.5 Numerical stability analysis of FAM 2D-RLS, 2D-NLMS & 2D-RLS for 16 bit quantized data 28

3.1 Two-way relay system with node capability 32
3.2 Two-way relay system with relay capability 36
3.3 pedA channel power delay profile 42
3.4 pedB channel power delay profile 42
3.5 vehA channel power delay profile 43
3.6 Frequency response of the channels 43
3.7 Dependence of channel estimation MSE and forgetting factor for FAM 2D-RLS 44
3.8 Convergence performance of FAM 2D-RLS and 2D-NLMS for estimating $\mathbf{h}^{(1)}$ in the case of a pedA channel 45
3.9 Convergence performance of FAM 2D-RLS and 2D-NLMS for estimating $\mathbf{g}^{(1)}$ in the case of a pedA channel 45
3.10 Convergence performance of FAM 2D-RLS and 2D-NLMS for estimating $\mathbf{h}^{(1)}$ in the case of a pedB channel 46
3.11 Convergence performance of FAM 2D-RLS and 2D-NLMS for estimating $\mathbf{g}^{(1)}$ in the case of a pedB channel 46
3.12 Convergence performance of FAM 2D-RLS and 2D-NLMS for estimating $\mathbf{h}^{(1)}$ in the case of a vehA channel 47

3.13 Convergence performance of FAM 2D-RLS and 2D-NLMS for estimating $\mathbf{g}^{(1)}$ in the case of a vehA channel 47
3.14 Convergence performance of FAM 2D-RLS & 2D-NLMS for estimating $\mathbf{h}^{(1,1)}$ at SNR = 30dB 49

4.1 Individual pedA channels $\mathbf{c}_n^{(1)}, \mathbf{c}_n^{(2)}$ 58
4.2 Overall pedA channel $\mathbf{g}_n$ 58
4.3 Bit allocation for RA and MA assuming perfect CSI for pedA channel 59
4.4 Energy allocation for RA and MA assuming perfect CSI for pedA channel . . . 60
4.5 Overall pedB channel $\mathbf{g}_n$ 61
4.6 Bit allocation for RA and MA assuming perfect CSI for pedB channel 61
4.7 Energy allocation for RA and MA assuming perfect CSI for pedB channel . . . 62
4.8 Overall vehA channel $\mathbf{g}_n$ 62
4.9 Bit allocation for RA and MA assuming perfect CSI for vehA channel 63
4.10 Energy allocation for RA and MA assuming perfect CSI for vehA channel . . . 63

List of Tables

1.1 A comparison of the computational cost in terms of the number of real addition, real multiplication and division required in the estimation of a single row of $\mathbf{G}_R$ or $\mathbf{G}_T$ 10

2.1 Computational Complexity per Iteration in terms of complex multiplication and complex addition for estimating a single row of weight matrix. 27

3.1 Computational Complexity of BFAM 2D-RLS per Iteration in terms of complex multiplication and complex addition 41

3.2 Performance of BFAM 2D-RLS for varying M in case of pedA channel 48

3.3 Performance of BFAM 2D-RLS for varying M in case of pedB channel 48

3.4 Performance of BFAM 2D-RLS for varying M in case of vehA channel 49

4.1 Performance of Levin-Campello RA/MA loading algorithm for varying values of MSE of pedA channel estimation for two-way relay 60

4.2 Performance of Levin-Campello RA/MA loading algorithm for varying values of MSE of pedB channel estimation for two-way relay 64

4.3 Performance of Levin-Campello RA/MA loading algorithm for varying values of MSE of vehA channel estimation for two-way relay 64

Chapter 1

Introduction

In the present scenario of communication systems, wireless devices like smart phones, laptops and tablets have become ubiquitous. These devices are used for applications that require large bandwidth, like High Definition (HD) video streaming. An obvious way to increase bandwidth would be to decrease the symbol rate. But this leads to Inter Symbol Interference (ISI) [1]. Multichannel modulation technique like Orthogonal Frequency Division Multiplexing (OFDM) [1] could be used to counter ISI. An introduction to OFDM is given in Appendix-A. The advantage of OFDM is that, by using Fast Fourier Transform (FFT) algorithm it can be implemented with low complexity. OFDM is the preferred modulation technique for the downlink of a Long Term Evolution (LTE) system [2]. Some of the latest research in wireless communication is in the field of relay based systems. Relay based communication finds wide scale use in LTE systems [3]. Relay based communication is also known as cooperative communication [4],[5]. In wireless communication, there might be scenarios where direct communication is not possible between source and destination due to the presence of large shadowing effect [1]. If a third party device or a dedicated relay is present between source and destination that has low shadowing effect, then this device can help in forwarding the data to destination[6]. In a wireless device like mobile phone the presence of multiple antennas is not feasible due to space constraint. Cooperative communication can be used to form diversity between the communicating nodes. This is known as virtual MIMO [7]. Another important use of relaying is to conserve the power in the communicating devices by helping to forward the data to the destination.

The simplest form of relay communication is the one-way relay[8]. But this technique is not spectrally efficient. Consider the case of a one hop communication system. In a one way relay, during the first time slot the sender node transmits data to relay while in the next time slot this data is forwarded to the destination. During these two time slots the destination node cannot transmit any information back to the source node. Thus a total of four time slots are required to complete the two way communication process while a non relay based system would have taken only two time slots. Thus the capacity of a one way relay is half that of a non relay system. In order to alleviate this problem, a two-way relaying scheme was proposed in [8]. In this case, both the communicating nodes send data simultaneously to the relay in the first time slot. In the next time slot the relay broadcasts this combined signal to the nodes. Thus the whole communication process would be completed in two time slots.

The high spectral efficiency of two-way relay systems and the advantages of OFDM can be combined to form an OFDM based two-way relay system [9]. In a coherent detection scheme the Channel State Information (CSI) should be known at the nodes. The channel frequency response of an OFDM system is usually correlated across both time and frequency [10]. The optimal estimator in the MMSE sense would be a 2D-Wiener filter [11]. But for implementing this filter the second order statistics of the channel are required [12]. In practice this would not be available at all times due to the time varying nature of the channel. Hence we introduce a low complexity 2D-RLS filter [13],[14] known as FAM 2D-RLS filter [15] for tracking the channel. An introduction to adaptive filter is given in Appendix-B. In order to reduce the number of preamble symbols required, the channel estimation is performed using the Decision Directed (DD) principle [16]. In DDCE, the detected data of the previous sample is used to find a Least Square (LS) estimate of the channel in the present time. In order to improve this estimate, we use the FAM 2D-RLS filter [17]. To find the LS estimate of the channel in a two-way relay system, the methods used in MIMO-OFDM systems [18] are required even in the case of single antenna scenario.

In multicarrier systems, if the channel is known at the relay then bit and power could be allocated based on the SNR of each subchannel [19]. More power and bit could be allocated to the subchannel with higher SNR. These algorithms are known as loading algorithms. Using bit and power loading we can increase the capacity or decrease the BER. FAM 2D-RLS based channel estimation scheme implemented at the node is used to obtain the combined channel,i.e. the overall channel between Source-Relay-Destination. In order to calculate the SNR, individual channels are required. Since our proposed channel estimation technique is in the frequency domain, we can easily obtain the individual channel from the estimate of the combined channel. The loading algorithm used in this thesis is the Levin-Campello algorithm [20]. But our frame work could be used to implement other loading algorithms [21],[22],[23] for two-way relay systems.

Hence in this thesis we propose and implement a FAM 2D-RLS based channel estimation for OFDM based two-way relay systems. The channel estimate is used to implement a loading algorithm for two-way relay system. The effect of channel estimation error on loading algorithm is also analyzed.

A list of acronyms and notations used in this thesis are provided in Appendix-C and Appendix-D respectively.

1.1 Publications

The following are the publications based on which this thesis is prepared,

1.1.1 Journal

1. A.Joy and V.Chakka,"Fast Array Multichannel 2D-RLS Based OFDM Channel Estimator", *Circuits, systems and signal processing*, Springer, Nov.2012
2. A.Joy and V.Chakka, "Adaptive Channel Estimation and Loading for OFDM Based Two-Way Relay Systems",submitted to *Circuits, systems and signal processing*, Springer

1.1.2 Conference

1. A.Joy and V.Chakka, "An IQR-RLS based transceiver filter for MIMO two-way relaying in frequency selective environment", in *proc. UKIWCWS 2010*, IIT Delhi, Dec.2010.
2. A.Joy and V. Chakka, " Performance comparison of LMS/NLMS based transceiver filters for MIMO two-way relaying scheme",in *proc. ICCSP 2011*, NIT Calicut, Feb. 2011.
3. A.Joy and V.Chakka,"An Affine projection algorithm based transceiver filter for MIMO two-way relaying scheme", in *proc. wireless VITAEE 2011*, Chennai, Mar. 2011.
4. A.Joy and V.Chakka, "AOFDM Based Two-Way Relay Systems", *proc. AICERA 2013*, Kerala, June 2013
5. A.Joy and V.Chakka, "Joint Adaptive Channel Estimation and Transceiver Design for Two-Way Relay Systems", *proc. ICACC 2013*, Kerala, Aug 2013
6. A.Joy and V.Chakka, "Low Complexity 2D Adaptive Channel Estimation for OFDM Based Two-Way Relay Systems", *proc. of NGMAST 2013*, Prague, Czech Republic, Sept 2013

1.2 Motivation

In this section we discuss the initial simulation work done by us on two-way relay systems which is the motivation for channel estimation and adaptive OFDM schemes implemented in the later chapters of this thesis. In this work it is assumed that the relay is capable of performing complex signal processing techniques. Hence we call this system a two-way relay with relay capability. Adaptive transceivers at the relay based on LMS, NLMS and Partial Rank Algorithm (PRA) [24] are designed and their performances are compared. This work is an extension of [25].

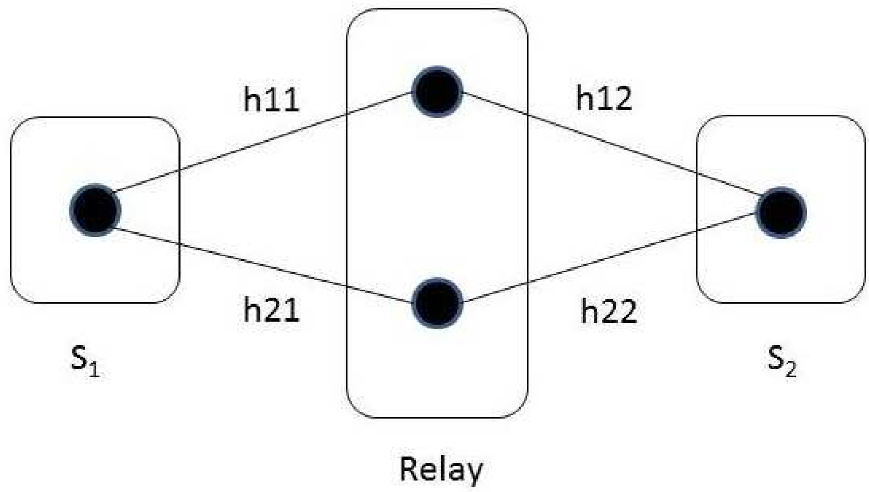

Figure 1.1: Two-Way relay with relay capability

1.2.1 System Model

In two-way relaying with relay capability, two nodes S_1 and S_2 are considered that communicates via an intermediate RS as shown in Figure 1.1. We assume that no direct path exists between the two nodes. The nodes S_1 and S_2 is assumed to have $M^{(1)}$ and $M^{(2)}$ antennas, respectively. For MIMO two way relaying it is assumed that $M^{(1)} = M^{(2)} = M$ and the RS is assumed to have $M_{RS} \geq M^{(1)} + M^{(2)} = 2M$ antennas. The data vector $\mathbf{x}^{(k)} = \left[x_1^{(k)}, \cdots, x_M^{(k)}\right]^T, k = 1, 2$ consists of data symbols $x_n^{(k)}, n = 1, \cdots, M$, transmitted from node S_k to the opposite node, where $[.]^T$ denotes the transpose of a matrix. The overall data vector is defined as a partitioned vector $\mathbf{x} = \left[\mathbf{x}^{(1)T} | \mathbf{x}^{(2)T}\right]^T$ with covariance matrix $\mathbf{R}_x = E\{\mathbf{x}\mathbf{x}^H\}$ where $E\{.\}$ and $[.]^H$ denote the expectation and conjugate transpose [26] respectively. The transmission phase consists of both the nodes transmitting to the RS at the same time. During this phase the data to be transmitted from S_1 and S_2 is processed using scalar transmit filters defined by matrices $\mathbf{Q}^{(1)}$ and $\mathbf{Q}^{(2)}$ respectively where $\mathbf{Q}^{(1)} = q^{(1)}\mathbf{I}_M$ and $\mathbf{Q}^{(2)} = q^{(2)}\mathbf{I}_M$. A scalar transmit filter is assumed because spatial filtering is done at the RS. The identity matrix of dimension M is defined as $\mathbf{I}_M$ .We assume that $E^{(1)}$ and $E^{(2)}$ are the maximum transmit energies of S_1 and S_2 respectively. Then the energy transmit constraint is given by $E\{\|q^{(k)}\mathbf{x}^{(k)}\|^2\} \leq E^{(k)},\ k = 1, 2$ where $\|.\|$ is the Euclidean norm of a vector. The overall scalar transmit filter $\mathbf{Q}$ is a block diagonal matrix with diagonal elements as $\mathbf{Q}^{(1)}$ and $\mathbf{Q}^{(2)}$. The channel is assumed to be flat fading and the channel matrix [25] between node $S_k, k = 1, 2$ and RS is denoted as $\mathbf{H}^{(k)}$. The overall channel matrix is defined as the partitioned matrix $\mathbf{H} = \left[\mathbf{H}^{(1)} | \mathbf{H}^{(2)}\right]$. The data received at the RS is called the receive vector and is denoted as $\mathbf{y}_{RS} = \mathbf{HQx} + \mathbf{n}_{RS}$. At RS the receive vector is passed through a spatial filter $\mathbf{G}$ leading to the RS transmit vector denoted as $\mathbf{x}_{RS}$. The RS transmit vector has to satisfy the constraint

$E\{\|\mathbf{x}_{RS}\|^2\} \leq E_{RS}$ where E_{RS} is the maximum transmit energy at the RS. During the receive phase, the RS transmit vector is simultaneously transmitted to S_1 and S_2. The scalar receive filter at S_1 and S_2 is $\mathbf{P} = P\mathbf{I}_{2M}$. A scalar receive filter is assumed because the spatial filtering is done at RS. It is assumed that the channel matrix is constant for two time slots. So the channel from RS to S_1 and S_2 can be considered to be $\mathbf{H}^T$. The RS after processing the data using the transceiver filter sends it simultaneously to S_1 and S_2. The estimate for data vector $\mathbf{x}_2$ at S_1 is denoted as $\hat{\mathbf{x}}_1$ and the estimate for data vector $\mathbf{x}_1$ at S_2 is denoted as $\hat{\mathbf{x}}_2$. The overall partitioned estimated data vector $\hat{\mathbf{x}} = [\hat{\mathbf{x}}_1|\hat{\mathbf{x}}_2]$ is given as $\hat{\mathbf{x}} = p(\mathbf{H}^T\mathbf{GHQx} + \mathbf{H}^T\mathbf{G}\mathbf{n}_{RS} + \mathbf{n}_R)$ where $\mathbf{n}_R$ is an AWGN with covariance matrix $\mathbf{R}_{n_R}$.

1.2.2 Linear Transceiver Filter

The linear transceiver filter [27] consists of the linear combination of three filters and is defined as, $\mathbf{G} = \mathbf{G}_T\mathbf{G}_\pi\mathbf{G}_R$ where $\mathbf{G}_T, \mathbf{G}_\pi$ and $\mathbf{G}_R$ are the transmit, mapping and receive filters respectively . The functionality of these filters are as explained as follows,

During the transmission phase, data is sent simultaneously from S_1 and S_2 to the RS. During the RS data processing phase, RS will estimate the transmitted data vector from the receive vector using the receive filter $\mathbf{G}_R$. This estimated data vector is denoted as a partitioned vector, $\hat{\mathbf{x}}_{RS} = \left[\hat{\mathbf{x}}_{RS}^{(1)T}|, \hat{\mathbf{x}}_{RS}^{(2)T}\right]^T$ with $\hat{\mathbf{x}}_{RS}^{(1)}$ being the estimate for $\mathbf{x}^{(1)}$ and $\hat{\mathbf{x}}_{RS}^{(2)}$ being the estimate of $\mathbf{x}^{(2)}$.The estimated data is then input to RS mapping filter. This is done so as to ensure that, S_1 is provided with an estimate of data vector $\mathbf{x}^{(2)}$ and S_2 is provided with estimate of data vector $\mathbf{x}^{(1)}$. The mapped data vector is then passed through the transmit filter $\mathbf{G}_T$ so as to produce the RS transmit vector $\mathbf{x}_{RS}$. The transmit filter separates the data to S_1 and S_2 before retransmission and performs self interference cancellation. Hence no CSI is required at S_1 and S_2. The use of transceiver filter at the RS reduces the processing load at the nodes S_1 and S_2. MMSE cost function for designing the receive filter is similar to that in [25]. The instantaneous scalar transmit and receive filter coefficients are obtained using Karush-Kuhn-Tucker (KKT) conditions [28] as,

$$q_i^{(k)} = \sqrt{\frac{E^{(k)}}{tr\{\mathbf{R}_{\mathbf{x},i}\}}}\, k = 1,2 \tag{1.1}$$

$$\mathbf{G}_T = \frac{1}{p} tr\left\{\left(\mathbf{H}^*\mathbf{H}^T + \frac{tr\{\mathbf{R}_{\mathbf{n}_R}\}}{E_{RS}}\mathbf{I}\right)^{-1}\right\}\mathbf{H}^*\mathbf{R}_{\hat{\mathbf{x}}_{RS,i}} \tag{1.2}$$

and,

$$p_i = \sqrt{\frac{tr\left\{\left(\mathbf{H}^*\mathbf{H}^T + \frac{tr\{\mathbf{R}_{\mathbf{n}_R}\}}{E_{RS}}\mathbf{I}\right)^{-1}\right\}^{-2}\mathbf{H}^*\mathbf{R}_{\hat{\mathbf{x}}_{RS,i}}}{E_{RS}}} \tag{1.3}$$

where $tr\{.\}$ is the trace of matrix, $[.]^*$ is the conjugate operation[5], q_i and p_i are the instantaneous coefficient of the scalar receive and transmit filters respectively while $\mathbf{R}_{\hat{x}_{RS},i}$ is the instantaneous covariance matrix.

LMS Transceiver Filter

<u>*LMS Receive Filter*</u>: This filter finds the optimal $\mathbf{G}_R$ by doing multiple recursions. The recursion equation is defined as,

$$\mathbf{G}_{R,i} = \mathbf{G}_{R,i-1} + \mu\left[\mathbf{x}_i - \mathbf{G}_{R,i-1}\mathbf{y}_{RS,i}\right]\mathbf{y}_{RS,i}^H \tag{1.4}$$

where i denotes the i^{th} iteration and μ is the step size which should be properly chosen so that filter converges. In adaptive filter terminology $\mathbf{y}_{rs}$ is the regression vector while $\mathbf{x}$ is the desired data vector [24]. The initial value of $\mathbf{G}_R$ is denoted as $\mathbf{G}_{R,-1}$ is assumed to be Φ_{2M} i.e. a null matrix of dimension $2M \times 2M$.
<u>*LMS Transmit Filter*</u>: This filter finds the optimal $\mathbf{G}_T$ by performing multiple iterations. The recursion equation is given as,

$$\mathbf{G}'_{T,i} = \mathbf{G}'_{T,i-1} + \mu[\hat{\mathbf{x}}_{RS,i} - \mathbf{G}'_{T,i-1}\mathbf{z}_i]\mathbf{z}_i^H \tag{1.5}$$

where $\hat{\mathbf{x}}_{RS,i} = \mathbf{G}_\pi\hat{\mathbf{x}}_{RS}$, $\mathbf{G}'_{T,i} = p_i\mathbf{H}^T\mathbf{G}_{T,i}^H$ and $\mathbf{z}_i = \hat{\mathbf{x}}_{RS,\pi} + \mathbf{G}^+\mathbf{n}_R$. The Moore-Penrose pseudoinverse [29] of $\mathbf{G}'_T$ is denoted as $\mathbf{G}^+$ and initial value of $\mathbf{G}'_T$ is Φ_{2M}.

NLMS Transceiver Filter

In LMS algorithm the update direction is a scaled version of the regression vector i.e. for e.g. in the case of (1.4) μ is scaled by $\mathbf{e}_i\mathbf{y}_{RS,i}^H$ where $\mathbf{e}_i = [\mathbf{x}_i - \mathbf{G}_{R,i-1}\mathbf{y}_{RS,i}]$ and is known as the a priori output estimation error. So the amount of change in $\mathbf{G}_{R,i-1}$ during an iteration will be proportional to the norm of the regression vector. This can have adverse effect on the performance of LMS algorithm in cases where the signal consists of large amplitude variations. In this type of signals large fluctuations in the norm of the regression vector is observed. In order to rectify this problem the step size μ is normalized by the squared norm of the regression vector. This algorithm is known as NLMS algorithm.

NLMS Receive Filter: The recursion equation is defined as ,

$$\mathbf{G}_{R,i} = \mathbf{G}_{R,i-1} + \frac{\mu}{\|\epsilon + \mathbf{y}_{RS,i}\|^2}[\mathbf{x}_i - \mathbf{G}_{R,i-1}\mathbf{y}_{RS,i}]\mathbf{y}_{RS,i} \tag{1.6}$$

where ϵ is a small non-zero value which is included so as to ensure that the step size does not go to infinity when the norm of the regression vector becomes zero.
NLMS Transmit Filter: The recursion equation is given as,

$$\mathbf{G}'_{T,i} = \mathbf{G}'_{T,i-1} + \frac{\mu}{\|\epsilon + \mathbf{z}_i\|^2}[\hat{\mathbf{x}}_{RS,i} - \mathbf{G}'_{T,i-1}\mathbf{z}_i]\mathbf{z}_i^H \tag{1.7}$$

where $\hat{\mathbf{x}}_{RS}$, $\mathbf{G}'_T$ and $\mathbf{z}_i$ are the same as in the case of LMS transmit filter.

PRA Transceiver Filter

PRA Receive Filter: This filter finds the optimal $\mathbf{G}_R$ by performing multiple iterations. Let $\mathbf{A} = \mathbf{G}_R^H$ and is of dimension $2M \times 2M$. The recursive update equation is given as,

$$\mathbf{a}_{n,i} = \mathbf{a}_{n,i-2M} + \mu\mathbf{Y}_{RS,i}[\epsilon\mathbf{I} + \mathbf{Y}_{RS,i}^H\mathbf{Y}_{RS,i}]^{-1}[\mathbf{X}_{n,i}^{(k)} - \mathbf{Y}_{RS,i}^H\mathbf{a}_{n,i-2M}] \tag{1.8}$$

where $n = 1 \cdots 2M$, $k = 1, 2$ and $\mathbf{a}_{n,i}$ is the n^{th} column of $\mathbf{A}$ at i^{th} iteration. $\mathbf{Y}_{RS,i} = [\mathbf{y}_{RS,i}, \mathbf{y}_{RS,i-1} \cdots \mathbf{y}_{RS,i-2M+1}]$,where $\mathbf{y}_{RS,i}$ is the receive vector at i^{th} iteration. The data vector at node S_k $k = 1, 2$ is $\mathbf{X}_{n,i}^{(k)} = [x_{n,i}^{(k)} \cdots x_{n,i-2M+1}^{(k)}]^H$. The regularization factor [24] is represented as ϵ and $\mathbf{I}$ is a unit vector of dimension $2M \times 2M$. The step size is denoted as μ. Note that in (1.8), the weight vector at i^{th} iteration depends on the weight vector at iteration $i - 2M$. Thus the weight vector is kept constant for $2M$ iterations. This is the difference between the standard Affine Projection Algorithm (APA) [24] and PRA.
PRA Transmit Filter: The optimal $\mathbf{G}_T$ in the least square sense is found using multiple recursions. Let, $\mathbf{G}'_T = p_i\mathbf{H}^T\mathbf{G}_T\mathbf{H}$ and $\mathbf{z}_i = \hat{\mathbf{x}}_{RS} + \mathbf{G}^{+}\mathbf{n}_R$ where $\mathbf{G}^{+}$ is Moore-Penrose pseudoinverse [29] of $\mathbf{G}'_T$. Finding optimal value of $\mathbf{G}_T$ is same as finding optimal value of G'_T. Define a matrix $\mathbf{B} = \mathbf{G}_T'^H$ which is of dimension $2M \times 2M$. The recursive update equation is given as,

$$\mathbf{b}_{n,i} = \mathbf{b}_{n,i-2M} + \mu\mathbf{Z}_i[\epsilon\mathbf{I} + \mathbf{Z}_i^H\mathbf{Z}_i]^{-1}[\hat{\mathbf{x}}_{RS,i}^{(k)} - \mathbf{Z}_i^H\mathbf{b}_{n,i-2M}] \tag{1.9}$$

where $n = 1 \cdots 2M$, $k = 1, 2$ and $\mathbf{b}_{n,i}$ is the n^{th} column of $\mathbf{B}$ at i^{th} iteration. $\mathbf{Z}_i = [\mathbf{z}_i, \mathbf{z}_{i-1} \cdots \mathbf{z}_{i-2M+1}]$. The data vector is $\hat{\mathbf{x}}_{RS,i}^{(k)} = \left[\hat{\mathbf{x}}_{RS,i}^{(k)} \cdots \hat{\mathbf{x}}_{RS,i-2M+1}^{(k)}\right]$.

1.2.3 Simulation Results

In this section MSE performance of the LMS, NLMS and PRA transceiver filter is compared using MATLAB simulations. It is assumed that S_1 and S_2 are equipped with $M = 1$ antenna

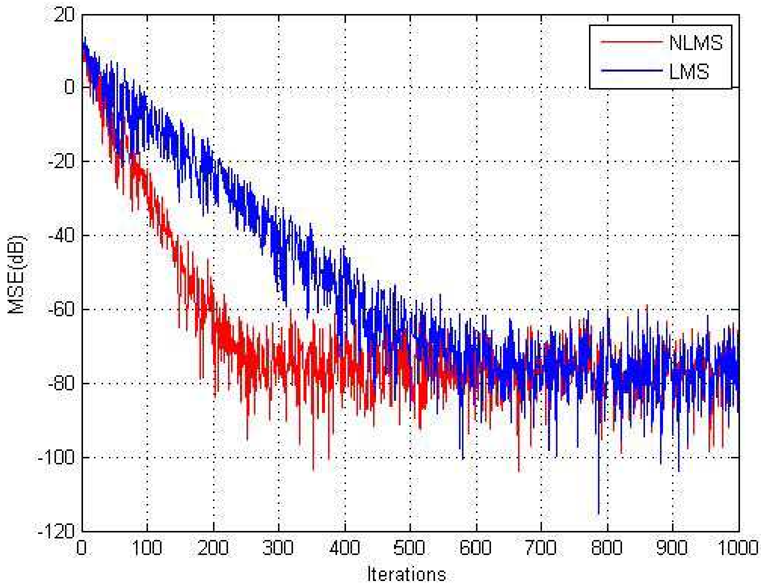

Figure 1.2: MSE performance of NLMS and LMS receive filter

each and RS is equipped with $M_{RS} = 2$ antennas. The data of S_1 and S_2 is QPSK modulated. The channel coefficients are spatially white and their amplitude is Rayleigh distributed.The experimental results are averaged over 100 runs.

In Figure 1.2,1.3 the performance comparison of MSE for LMS and NLMS for receive and transmit filter is shown respectively. Using the results in [24] the step size of the NLMS transmit and receive filter are both considered to be 0.2.The ϵ value is 0.001. In the case of LMS transceiver filter the step size is considered to be 0.01. It is seen that NLMS transceiver filter has a faster rate of convergence compared to LMS transceiver filter. This can be attributed to the fact that NLMS algorithm is obtained as a stochastic gradient approximation to Newton's method while LMS is a stochastic gradient approximation of the steepest descent algorithm . It is shown in [24] that the Newton's method has a superior convergence rate compared to that of the steepest descent algorithm. In the case of receive filter the NLMS algorithm converges at 200 iterations while it takes about 400 iterations for the LMS algorithm to converge. For the case of transmit filter it is seen that the NLMS algorithm converges at 100 iterations while LMS converges at 280 iterations. For the case of receive filter the step size is taken as 0.2 while for that of transmit filter it is taken as 0.1.

In Figure 1.4 it is seen that PRA converges around 40 iterations but the NLMS algorithm converges around 200 iterations. In Figure 1.5 it is seen that PRA based transmit filter outperforms the NLMS based filter. So in a fast varying channel environment a PRA based transceiver filter could be used. In Table 1.1, the computational complexity of different adaptive transceiver filters are shown. It is observed that PRA has a computational complexity of $O(M^2)$ while that

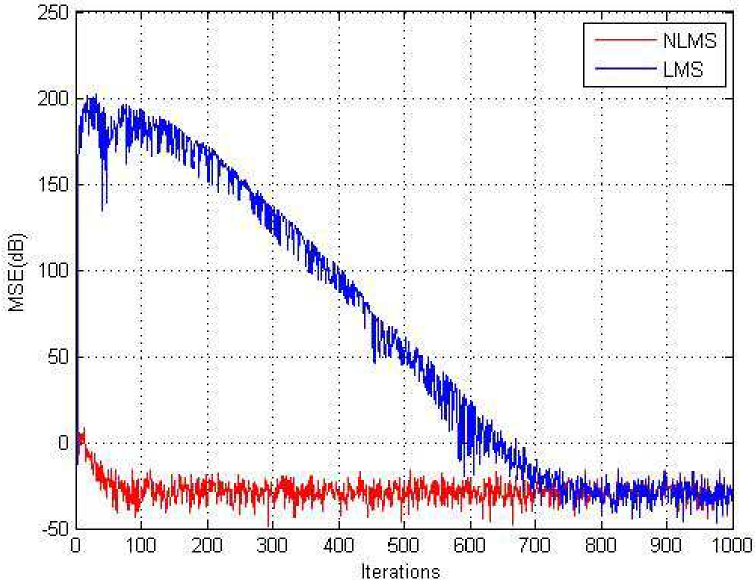

Figure 1.3: MSE performance of NLMS and LMS transmit filter

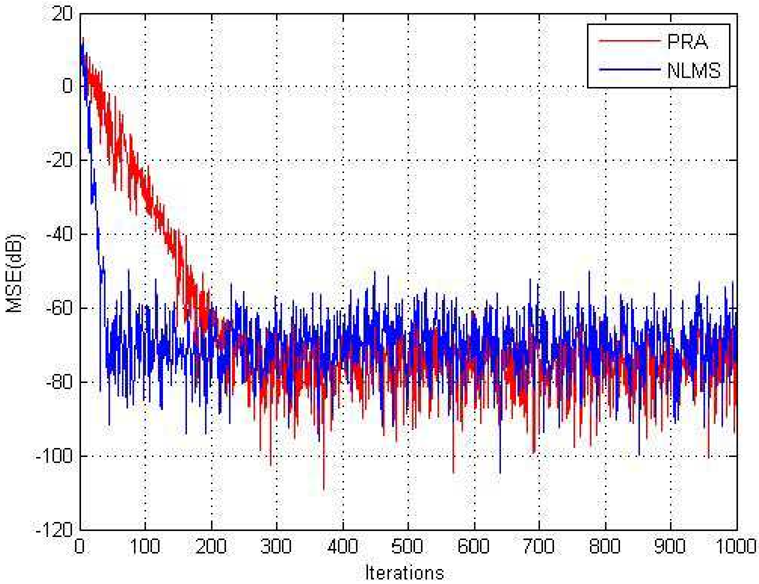

Figure 1.4: MSE performance of PRA and NLMS receive filter

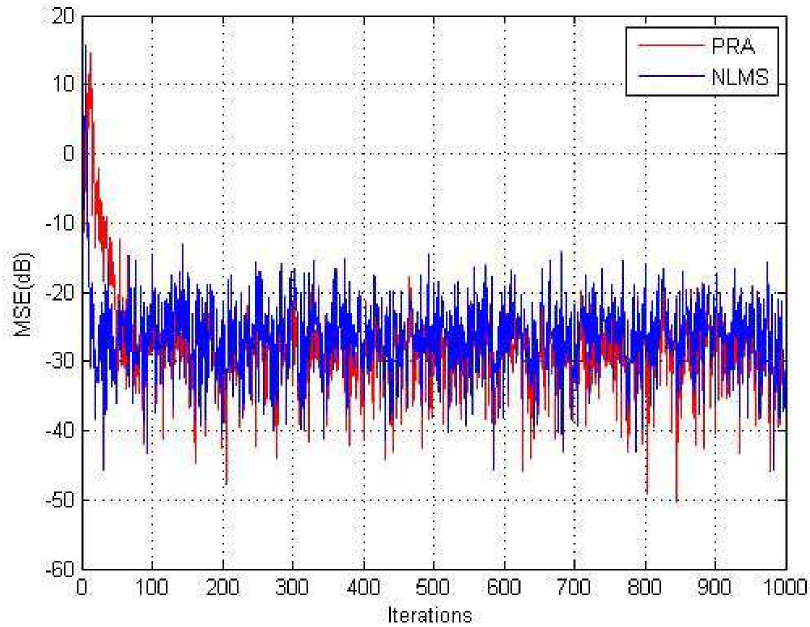

Figure 1.5: MSE performance of PRA and NLMS transmit filter

Algorithm	$\times$	$+$
PRA	$8M^2 + 4M + 1$	$8M^2 + 16M$
NLMS	$20M + 3$	$20M$
LMS	$16M + 2$	$16M$

Table 1.1: A comparison of the computational cost in terms of the number of real addition, real multiplication and division required in the estimation of a single row of $\mathbf{G}_R$ or $\mathbf{G}_T$

of NLMS and LMS is $O(M)$. In practical scenario where the antennas in the relay and source are less, the computational cost of APA and NLMS/LMS are comparable.

1.2.4 Open Problems

1. The simulations in this section assumed that the channel is flat . But this is not valid in practical scenario, due to the presence of multipath.

2. The multipath channel could be converted to flat parallel channels with the use of OFDM. Hence the performance of OFDM based two-way relay systems should be analyzed

3. It is assumed in this section that the relay is capable of complex signal processing tasks. But this is only valid in the presence of dedicated relays. A more practical scenario would be a third party device acting as a relay. In this case the signal processing tasks should be implemented at the nodes. Hence a two-way relay with node capability which uses OFDM should be proposed and its performance analyzed

4. In the above simulations it is assumed that CSI is available and is time invariant. But this is not a valid assumption in real scenario. Hence a low cost 2D adaptive channel estimation based filter should be proposed for time varying channel

5. If CSI is available at node then we can perform adaptive OFDM [30] to improve the bit rate and power efficiency

1.3 Thesis contribution

This thesis solves all the open problems mentioned in the previous section.

1. In Chapter-2, we propose a low complexity 2D adaptive filter called FAM 2D-RLS which is used for estimating an OFDM channel. The complexity of this filter is similar to the existing 2D-NLMS [31] filter but has same convergence rate as that of 2D-RLS [13],[14] filters. Steady state analysis of this filter is also performed.

2. In Chapter-3 we use FAM 2D-RLS filter for estimating the OFDM channel of two-way relay systems with node capability and relay capability respectively.

3. In Chapter-4 we use the channel estimate to perform bit and power loading so as to increase the overall system capacity and power efficiency for a two-way relay system

Chapter 2

Fast Array Multichannel 2D-RLS Based OFDM Channel Estimator

2.1 Introduction

In the present scenario of wireless communication, OFDM is a prominent modulation technique due to its ability to provide high data rate, robustness to Inter Symbol Interference (ISI) and ease of implementation [32]. For coherent detection of data, the channel information is required at the receiver [1]. There is abundant research material available on the topic of OFDM channel estimation [33]. In [34] channel estimation based on time domain channel statistics have been introduced. It was shown that Minimum Mean Square Error (MMSE) estimator has lesser Symbol Error Rate (SER) compared to Least Square (LS) estimator especially in the case of low Signal to Noise ratio (SNR). In [35], the performance of LS estimation is improved by performing Inverse Discrete Fourier Transform(IDFT) of the Channel Frequency Response (CFR) and then replacing all elements greater than the multipath length by zeros. This is called the significant path capture method. In [10], a robust MMSE channel estimator which is insensitive to the channel statistics have been proposed. In [36], an OFDM channel estimator for time varying channels have been introduced, where the time variation of the channel is captured by a basis expansion model. In [37], a channel estimation technique for the case of insufficient Channel Prefix (CP) is studied. Channel estimation for various OFDM pilot placement is studied in [38]. Preamble based OFDM channel estimation scheme for the use in two-way relay network is discussed in [39]. But all the methods mentioned above assume that the true channel statistics is available with the receiver at all times. But this is impractical, especially in the case of mobile devices where the channel statistics might vary with time. Thus in [40] an adaptive channel estimator for OFDM based on Normalized Least Mean Square (NLMS) and RLS algorithms have been proposed. This filter estimates the channel by making use of the correlation of the channel coefficients across time. An adaptive channel estimation scheme based on Kalman fil-

ter is implemented in [41]. In [1] it is shown that channel coefficients are correlated across both time and frequency. So the optimum estimator in the MMSE sense would be the 2D-Wiener filter [42]. A 2D-RLS channel estimator has been implemented in [13]. In order to combat the numerical instability inherent in the classical RLS algorithm [24], an array based 2D-RLS was implemented in [14]. But the RLS algorithms of [13] and [14] have a computational cost of $O(M^2)$ in the case of an M order filter. This makes the 2D-RLS filter virtually unusable for channel estimation for OFDM in the frequency domain. In order to reduce this complexity, a 2D-NLMS [31] filter could be implemented. But generally NLMS has a poorer convergence rate compared to the RLS algorithm [24]. So it might not track fast varying channels.
In this chapter we design a 2D adaptive filter for OFDM channel estimation which has similar complexity as 2D-NLMS i.e. $O(M)$, convergence property of the classical 2D-RLS and having numerical stability in finite precision. In [24], it is shown that by making use of the shift structure of the input data vector, the computational cost of classical RLS is reduced while providing the same convergence rate. In this chapter we show the similarity between 2D adaptive estimation and multichannel adaptive filter. Thus fast 2D-RLS adaptive filter could be used for OFDM channel estimation. The performance is evaluated using MATLAB simulations.

2.2 2D OFDM Channel Estimation

Consider the length of the channel to be L. If the channel prefix $CP \geq L - 1$, then ISI is eliminated. Also assume that the channel is constant for at least one OFDM symbol duration. Then the OFDM system can be represented by K parallel Gaussian channels, where K is the IFFT size for generating the OFDM symbols. If $\mathbf{x}_n$ is the $K \times 1$ transmit data vector, then the received data vector after performing CP removal and K-point DFT is,

$$\mathbf{y}_n = \mathbf{X}_n \mathbf{h}_n + \mathbf{w}_n \tag{2.1}$$

where $\mathbf{X}_n = diag\{\mathbf{x_n}\}$ and $\mathbf{w}_n$ is AWGN noise in the frequency domain. The channel frequency response of dimension $K \times 1$ is defined as,

$$\mathbf{h}_n = [h(n,0)^*, h(n,1)^*, \cdots, h(n,K-1)^*]^H \tag{2.2}$$

During the initial phase of data transmission(training phase), OFDM symbols $\mathbf{x}_n^p$ called preambles which are known at the receiver is sent. Then the LS estimate of the channel frequency response is,

$$\bar{\mathbf{h}}_\mathbf{n} = (\mathbf{X}_n^p)^{-1} \mathbf{y}_n \tag{2.3}$$

Since the channel is assumed to be slowly varying, the OFDM channel estimator can be assumed to work based on the Decision Directed (DD) principle introduced in [16]. The advantage of using DD estimator is that, the detected OFDM symbols can be used as preamble for estimating the channel frequency response. If $\bar{\mathbf{x}}_{n+1}$ is the LS estimate of the $(n+1)^{th}$ transmitted data, then the DD-OFDM channel estimator works as follows,

$$\bar{\mathbf{x}}_{n+1} = \hat{\mathbf{H}}_n^{-1}\mathbf{y}_{n+1} \tag{2.4}$$

where $\hat{\mathbf{H}}_n^{-1} = diag\{\hat{\mathbf{h}}_n\}$. The vector $\hat{\mathbf{h}}_n$ is the estimate of the channel coefficient obtained by performing a 2D filtering on the LS channel estimate. The vector $\bar{\mathbf{x}}_{n+1}$ is then passed through a decision device in order to detect the transmit data vector. Assuming that there is no error in the detected data, the LS estimate of $(n+1)^{th}$ channel coefficient vector is obtained as,

$$\bar{\mathbf{h}}_{n+1} = \mathbf{X}_{n+1}^{-1}\mathbf{y}_{n+1} \tag{2.5}$$

Let $\bar{\mathbf{u}}_n$ be an $NK \times 1$ vector that contains the LS channel estimate for the past N time samples. It is defined as,

$$\bar{\mathbf{u}}_n = \left[\bar{h}^*(n,0)\cdots\bar{h}^*(n,K-1)\cdots\bar{h}^*(n-N+1,0)\cdots\bar{h}^*(n-N+1,K-1)\right]^H \tag{2.6}$$

Assuming that $\mathbf{h}_n$ is the true channel frequency response vector at time n of dimension $K \times 1$, our aim is to design an adaptive filter to solve the following exponentially weighted regularized least square problem [24],[14],

$$arg\ \underset{\bar{\mathbf{W}}_n}{min}\ \left[\lambda^{n+1}\bar{\mathbf{W}}_n\mathbf{\Pi}\bar{\mathbf{W}}_n^H + \sum_{i=0}^{n}\lambda^{n-i}\|\mathbf{h}_n - \hat{\mathbf{h}}_n\|^2\right] \tag{2.7}$$

where the estimate of the channel at time n is,

$$\hat{\mathbf{h}}_n = \bar{\mathbf{W}}_n\bar{\mathbf{u}}_n \tag{2.8}$$

and $\bar{\mathbf{W}}_n$ is of dimension $K \times NK$. The regularization factor is $\mathbf{\Pi} = \delta\mathbf{I}_{NK\times NK}$. The method of selecting the regularization parameter δ and the forgetting factor λ is discussed in the later sections.

It is observed in (2.7), that in order to design the adaptive filter, we require the true channel frequency response. The true channel frequency response acts as the reference signal to the adaptive filter. But satisfying this condition is impossible. Hence we need to obtain an estimate of the reference signal vector. During the training period of the adaptive filter the significant path capture technique is performed on $\bar{\mathbf{h}}_n$ in order to obtain a noise reduced estimate of $\mathbf{h}_n$ [5].

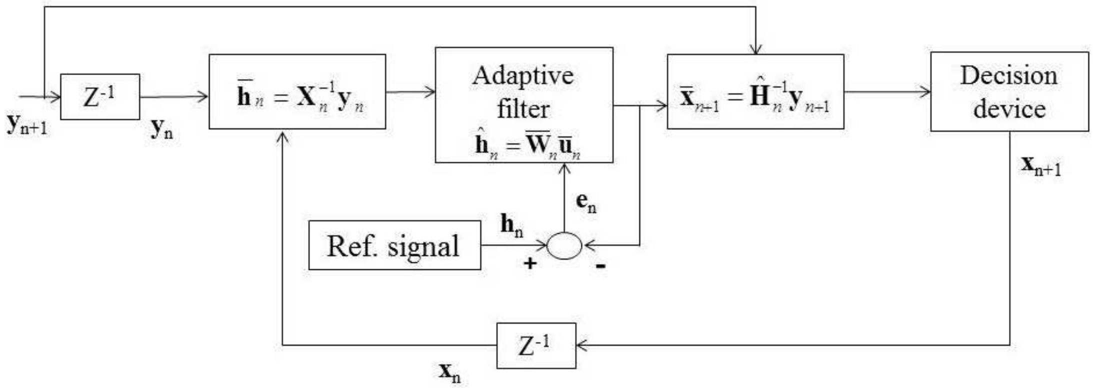

Figure 2.1: Adaptive DD channel estimator for OFDM system

During significant path capture method, K-point DFT of $\bar{\mathbf{h}}_n$ is performed. Then consider the L largest coefficients while discarding the rest. Perform a K-point DFT on the L coefficients to obtain a noise reduced estimate of $\mathbf{h}_n$. After the convergence of the adaptive filter, the reference signal vector can be obtained by assuming $\mathbf{h}_n \approx \hat{\mathbf{h}}_{n-1}$. This is possible since the channel is assumed to be slowly varying. The block diagram of the adaptive DD-OFDM channel estimator is shown in Figure 2.1.

2.3 Fast Array Multichannel 2D-RLS

The computational cost for estimating a single row of weight matrix $\bar{\mathbf{W}}_n$ using classical algorithm based 2D-RLS [13],[24] is $O((NK)^2)$ where NK is the order of the filter, e.g for the case of 256 point OFDM, assuming time correlation for 3 time samples i.e. $N = 3$, $K = 256$ the cost is $O(768^2)$. But by making use of the shift structure of the input vector, the computational cost can be reduced to $O(NK)$. In [43], a fast array based RLS has been used to design an adaptive transmultiplexer while in [44], a block based fast RLS has been used in radar imaging application.

In this chapter we extend the 1D fast multichannel RLS of [43],[24],[44] into 2D and apply it for the channel estimation of OFDM systems. In order to bring in the notion of shift structure, we rewrite $\bar{\mathbf{u}}_n$ as,

$$\mathbf{u}_n = \left[\bar{h}^*(n,0)\cdots\bar{h}^*(n-N+1,0)\cdots\bar{h}^*(n,K-1)\cdots\bar{h}^*(n-N+1,K-1)\right]^H \quad (2.9)$$

and define,

$$\mathbf{c}_{n,k} = \left[\bar{h}^*(n,k),\bar{h}^*(n-1,k)\cdots,\bar{h}^*(n-N+1,k)\right]^H, \quad k = 0\cdots K-1 \quad (2.10)$$

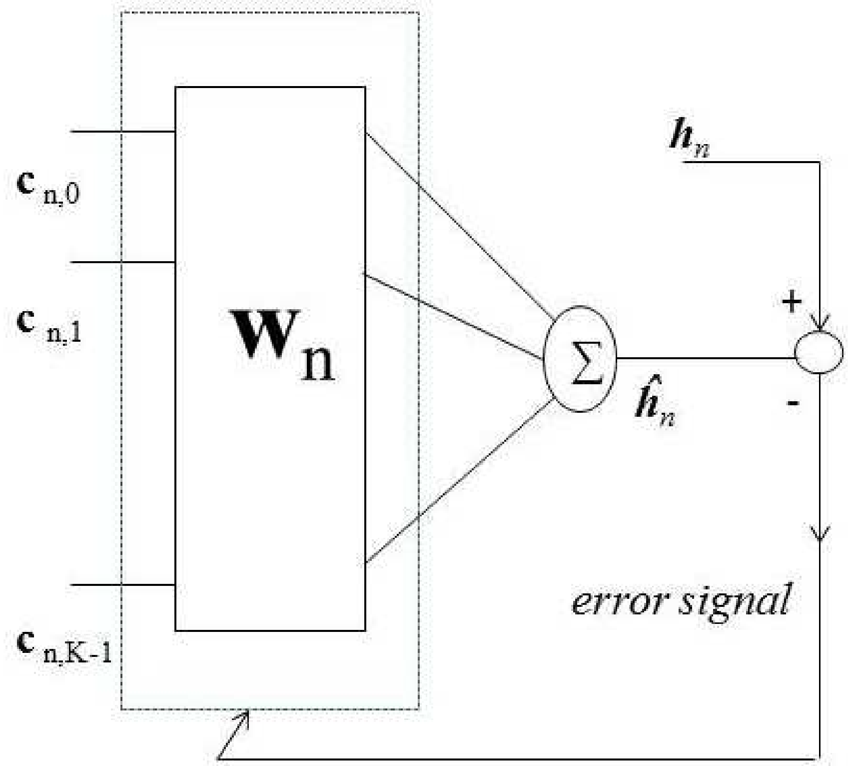

Figure 2.2: 2D OFDM Adaptive Channel Estimator as a Multichannel Adaptive Filter

Even though $\mathbf{u}_n$ does not have a shift structure in the real sense it is seen that each subvector $\mathbf{c}_{n,k}$ has a shift structure. So we can consider each $\mathbf{c}_{n,k}$ as the input vector of K parallel N-tap adaptive filters. This is equivalent to stating that individual subcarrier of the OFDM symbol is provided with an N tap adaptive filter and they work in parallel to estimate $\mathbf{h}_n$ as shown in Figure 2.2. Then (2.8) is rewritten as,

$$\hat{\mathbf{h}}_n = \mathbf{W}_n \mathbf{u}_n \tag{2.11}$$

where $\mathbf{W}_n$ is obtained by rearranging the terms in $\bar{\mathbf{W}}_n$ in accordance with the changes that where made in $\bar{\mathbf{u}}_n$, to obtain $\mathbf{u}_n$. Hence we have shown that an adaptive OFDM channel estimation that makes use of the correlation of channel coefficients in frequency and time can be converted to multichannel adaptive filtering problem with time shift structure.

2.3.1 Adaptive Filter as an Iterative Equation Solver

The aim of an adaptive filter is to solve the mean square optimization problem iteratively. Adaptive filter also tracks the statistical changes in the system. All the standard literature considers the various algorithms of adaptive filtering to be unrelated. But in [45] it is shown than this is not the case. It is proved that any adaptive filter can be considered to be an iterative equation solver of the Wiener-Hopf equation. The advantage of viewing an adaptive filter in this perspective is that all the different adaptive filter algorithms could be designed and analyzed using a unified framework. The steady state analysis of the RLS filter designed in this chapter is done based on this principle. Hence our approach can be used to analyze some of the other common adaptive filters like Affine projection or LMS. This unified framework for analysis is called the energy conservation method [24].

Consider the following optimization problem,

$$arg \underset{\bar{\mathbf{W}}_n}{min} \|\mathbf{h}_n - \mathbf{W}_n \mathbf{u}_n\|^2 \tag{2.12}$$

where $\mathbf{h}_n, \mathbf{W}_n$ and $\mathbf{u}_n$ have the same meaning as in the previous section. In case the second order statistics of the signals are available, then we can find the optimal weight matrix $\mathbf{W}^o$ by solving the Wiener Hopf equation defined as

$$\mathbf{W}^o \mathbf{R}_{\mathbf{uu}} = \mathbf{R}_{\mathbf{hu}} \tag{2.13}$$

where $\mathbf{R}_{\mathbf{uu}} = E\left[\mathbf{u}\mathbf{u}^H\right]$ and $\mathbf{R}_{\mathbf{hu}} = \mathbf{E}\left[\mathbf{h}\mathbf{u}^{\mathbf{H}}\right]$ are the autocovariance matrix of $\mathbf{u}$ and cross-covariance matrix of $\mathbf{h}$,$\mathbf{u}$ respectively. This linear equation could be solved by direct or iterative methods [46]. A direct method finds the solution in a prescribed finite number of steps while iterative methods find an approximate solution at each iteration and after a finite number of iterations might converge to the true solution. All the versions of the Gaussian elimination methods are direct methods [29] while some of the common iterative methods are Jacobi's method, Gauss-Siedel method and successive over relaxation method. All the iterative algorithms can be unified by the notion of splitting of the coefficient matrix. This is known as the Richardson's method [47]. Let's consider the Wiener-Hopf equation in (2.13). Assume that $\mathbf{R}_{\mathbf{uu}}$ can be splitted as,

$$\begin{aligned} \mathbf{R}_{\mathbf{uu}} &= \mathbf{M} - \mathbf{N} \\ &= \mathbf{M} - (\mathbf{M} - \mathbf{R}_{\mathbf{uu}}) \end{aligned} \tag{2.14}$$

where $\mathbf{M}$ is a non singular matrix. Substituting this in (2.13) and forming an iterative equation,

$$\begin{aligned} &\mathbf{W}_n = \mathbf{W}_{n-1}(\mathbf{M} - \mathbf{R}_{\mathbf{uu}})\mathbf{M}^{-1} + \mathbf{R}_{\mathbf{hu}}\mathbf{M}^{-1} \\ &= \mathbf{W}_{n-1} + (\mathbf{R}_{\mathbf{hu}} - \mathbf{W}_{n-1}\mathbf{R}_{\mathbf{uu}})\mathbf{M}^{-1} \end{aligned} \tag{2.15}$$

The same iterative equation can be obtained if we try to solve the equation

$$\mathbf{W}^o \mathbf{R}_{\mathbf{uu}} \mathbf{M}^{-1} = \mathbf{R}_{\mathbf{hu}} \mathbf{M}^{-1} \tag{2.16}$$

The number of iterations required for the equation to converge to the actual solution depends on the selection of $\mathbf{M}^{-1}$.The matrix $\mathbf{M}^{-1}$ is known as the preconditioner [48]. If we select

$\mathbf{M}^{-1} = \mathbf{R}_{\mathbf{uu}}^{-1}$, then the solution is obtained in a single iteration. But this negates the purpose of using an iterative equation. The precoder should be selected such that the solution can be solved easily. In 2.15 it is observed that the iterations can be performed only if $\mathbf{R}_{\mathbf{uu}}$ and $\mathbf{R}_{\mathbf{hu}}$ is known. But in real life the true autocovariance and cross covariance will not be available. Hence an approximation should be made. A good approximation of the covariance matrix are,

$$\mathbf{R}_{\mathbf{hu},n} \approx \frac{1}{j-i+1}\sum_{k=i}^{j}\mathbf{h}_k\mathbf{u}_k^H \tag{2.17}$$

and

$$\mathbf{R}_{\mathbf{uu},n} \approx \frac{1}{j-i+1}\sum_{k=i}^{j}\mathbf{u}_k\mathbf{u}_k^H \tag{2.18}$$

These equations could be written in matrix form as,

$$\begin{aligned}\mathbf{R}_{\mathbf{hu},n} &\approx \tfrac{1}{j-i+1}\begin{bmatrix}\mathbf{h}_i\\ \vdots \\ \mathbf{h}_j\end{bmatrix}\begin{bmatrix}\mathbf{u}_i^H \cdots \mathbf{u}_j^H\end{bmatrix} \\ &\approx \tfrac{1}{j-i+1}\mathbf{d}_n\mathbf{x}_n^H\end{aligned} \tag{2.19}$$

and

$$\begin{aligned}\mathbf{R}_{\mathbf{uu},n} &\approx \tfrac{1}{j-i+1}\begin{bmatrix}\mathbf{u}_i\\ \vdots \\ \mathbf{u}_j\end{bmatrix}\begin{bmatrix}\mathbf{u}_i^H \cdots \mathbf{u}_j^H\end{bmatrix} \\ &\approx \tfrac{1}{j-i+1}\mathbf{x}_n\mathbf{x}_n^H\end{aligned} \tag{2.20}$$

Using these approximations we can rewrite the Wiener Hopf equation in (2.13) as,

$$\mathbf{W}_n\mathbf{x}_n\mathbf{x}_n^H = \mathbf{d}_n\mathbf{x}_n^H \tag{2.21}$$

After using the Richardson splitting method we obtain,

$$\mathbf{W}_n = \mathbf{W}_{n-1} + (\mathbf{d}_n\mathbf{x}_n^H - \mathbf{W}_{n-1}\mathbf{x}_n\mathbf{x}_n^H)\mathbf{M}^{-1} \tag{2.22}$$

The various adaptive filters is defined by i, j and $\mathbf{M}^{-1}$. The NLMS filter as n^{th} iteration is defined by selecting $i = j = n$ and $\mathbf{M}^{-1} = \frac{\mu\mathbf{I}}{\|\mathbf{u}_n\|^2}$, where μ is the step size [24]. Hence the

NLMS weight update equation is,

$$\mathbf{W}_n = \mathbf{W}_{n-1} + \frac{\mu \mathbf{I}}{\|\mathbf{u}_n\|^2} \left[\mathbf{h}_n - \mathbf{W}_{n-1}\mathbf{u}_n\right] \mathbf{u}_n^H. \tag{2.23}$$

The exponentially weighted regularized RLS is defined by selecting $i = j = n$ and $\mathbf{M}_n^{-1} = \left[\lambda^{n+1}\Pi + \mathbf{u}_n\Lambda_n\mathbf{u}_n^H\right]^{-1}$ where $\Lambda_n = diag\{\lambda^n, \lambda^{n-1}, \cdots, 1\}$ and λ is the forgetting factor. It can be observed that $\mathbf{M}^{-1}$ selected for RLS is a better approximation of $\mathbf{R}_{\mathbf{uu}}$ than that of NLMS . This is the reason for the fast convergence of RLS compared to NLMS. It is observed in (2.22) that every iteration requires the calculation of the $\mathbf{M}^{-1}$. This requires a computational complexity of $O(NK)^3$ where $\mathbf{M}$ is of dimension $NK \times NK$. In order to reduce the computational complexity, $\mathbf{M}^{-1}$ can be be calculated recursively using the Woodsbury matrix identity [28]. Considering $\mathbf{M}^{-1} = \mathbf{P}_n$ the weight update equation is,

$$\mathbf{W}_n = \mathbf{W}_{n-1} + \mathbf{e}_n\mathbf{u}_n^H\mathbf{P}_n \tag{2.24}$$

where $\mathbf{e}_n = \mathbf{h}_n - \mathbf{w}_{n-1}\mathbf{u}_n$ is the a priori output estimation error [24].

2.3.2 Derivation of the FAM 2D-RLS

The derivation of the FAM 2D-RLS is based on modifying the classical RLS algorithm by making use of the shift structure of the input data. Hence we reproduce the 2D-RLS derived in [13],

$$\begin{aligned}
\mathbf{g}_n &= \lambda^{-1}\mathbf{P}_{n-1}\mathbf{u}_n\gamma(n), \quad (NK \times 1) && (2.25)\\
\gamma(n) &= \frac{1}{1+\lambda^{-1}\mathbf{u}_n^H\mathbf{P}_{n-1}\mathbf{u}_n}, \quad (1 \times 1) && (2.26)\\
\mathbf{P}_n &= \mathbf{P}_{n-1} - \frac{\mathbf{g}_n\mathbf{g}_n^H}{\gamma(n)}, \quad (NK \times NK) && (2.27)\\
\mathbf{e}_n &= \mathbf{h}_n - \hat{\mathbf{h}}_n, \quad (K \times 1) && (2.28)\\
\mathbf{W}_n &= \mathbf{W}_{n-1} + \mathbf{e}_n\mathbf{g}_n^H, \quad (K \times NK) && (2.29)
\end{aligned}$$

where $\mathbf{g}_n$ is the gain vector , $\gamma(n)$ is the conversion factor and $\mathbf{P}_n$ is the instantaneous covariance matrix. Due to the time shift structure in $\mathbf{u}_n$ we can relate it with $\mathbf{u}_{n-1}$ as ,

$$\left[\bar{h}(n,0)^*, \mathbf{c}_{n-1,0}^H, \bar{h}(n,1)^*, \mathbf{c}_{n-1,1}^H, \cdots, \bar{h}(n,K-1)^*, \mathbf{c}_{n-1,K-1}^H\right]^H =$$

$$\left[\mathbf{c}_{n,0}^H, \bar{h}(n-N,0)^*, \mathbf{c}_{n,1}^H, \bar{h}(n-N,1)^*, \cdots, \mathbf{c}_{n,K-1}^H, \bar{h}(n-N,K-1)^*\right]^H \tag{2.30}$$

The gain vector is partitioned as,

$$\mathbf{g}_n = \left[\mathbf{g}_n^{(0)H}\ \mathbf{g}_n^{(1)H}\ \ldots\ \mathbf{g}_n^{(K-1)H}\right]^H,\ (NK \times 1) \tag{2.31}$$

and the covariance matrix is partitioned as,

$$\mathbf{P}_n = \begin{bmatrix} \mathbf{P}_n^{(0,0)}, & \cdots, & \mathbf{P}_n^{(0,K-1)} \\ \vdots & \ddots & \vdots \\ \mathbf{P}_n^{(0,K-1)}, & \cdots, & \mathbf{P}_n^{(K-1,K-1)} \end{bmatrix} \tag{2.32}$$

where each $\mathbf{g}_n^{(k)}$ is of dimension $N \times 1$ and $\mathbf{P}_n^{(l,k)}$ is of dimension $N \times N$. In order to bring in the time shift structure relation of (2.30) into the RLS algorithm, we will modify (2.25) as,

$$\gamma^{-1}(n) \begin{bmatrix} \mathbf{g}_n^{(0)} \\ 0 \\ \mathbf{g}_n^{(1)} \\ 0 \\ \vdots \\ 0 \\ \mathbf{g}_n^{(K-1)} \\ 0 \end{bmatrix} =$$

$$\lambda^{-1} \begin{bmatrix} \begin{bmatrix} \mathbf{P}_{n-1}^{(0,0)} & \mathbf{0}_{N\times 1} \\ \mathbf{0}_{1\times N} & 0 \end{bmatrix} & \cdots & \begin{bmatrix} \mathbf{P}_{n-1}^{(0,K-1)} & \mathbf{0}_{N\times 1} \\ \mathbf{0}_{1\times N} & 0 \end{bmatrix} \\ \vdots & \ddots & \vdots \\ \begin{bmatrix} \mathbf{P}_{n-1}^{(K-1,0)} & \mathbf{0}_{N\times 1} \\ \mathbf{0}_{1\times N} & 0 \end{bmatrix} & \cdots & \begin{bmatrix} \mathbf{P}_{n-1}^{(K-1,K-1)} & \mathbf{0}_{N\times 1} \\ \mathbf{0}_{1\times N} & 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \bar{h}(n,0) \\ \mathbf{c}_{n-1,0} \\ \vdots \\ \bar{h}(n,K-1) \\ \mathbf{c}_{n-1,K-1} \end{bmatrix} \tag{2.33}$$

Similarly we can write,

$$\gamma^{-1}(n-1) \begin{bmatrix} 0 \\ \mathbf{g}_{n-1}^{(0)} \\ 0 \\ \mathbf{g}_{n-1}^{(1)} \\ 0 \\ \vdots \\ 0 \\ \mathbf{g}_n^{(K-1)} \end{bmatrix} =$$

$$\lambda^{-1}\begin{bmatrix}\begin{bmatrix}0 & \mathbf{0}_{1\times N}\\ \mathbf{0}_{N\times 1} & \mathbf{P}_{n-2}^{(0,0)}\end{bmatrix} & \cdots & \begin{bmatrix}0 & \mathbf{0}_{1\times N}\\ \mathbf{0}_{N\times 1} & \mathbf{P}_{n-2}^{(0,K-1)}\end{bmatrix}\\ \vdots & \ddots & \vdots \\ \begin{bmatrix}0 & \mathbf{0}_{1\times N}\\ \mathbf{0}_{N\times 1} & \mathbf{P}_{n-2}^{(0,K-1)}\end{bmatrix} & \cdots & \begin{bmatrix}0 & \mathbf{0}_{1\times N}\\ \mathbf{0}_{N\times 1} & \mathbf{P}_{n-2}^{(K-1,K-1)}\end{bmatrix}\end{bmatrix}\begin{bmatrix}\bar{h}(n,0)\\ \mathbf{c}_{n-1,0}\\ \vdots \\ \bar{h}(n,K-1)\\ \mathbf{c}_{n-1,K-1}\end{bmatrix} \tag{2.34}$$

subtracting (2.34) from (2.33) and utilizing (2.30), we obtain,

$$\gamma^{-1}(n)\begin{bmatrix}\mathbf{g}_n^{(0)}\\ 0\\ \mathbf{g}_n^{(1)}\\ 0\\ \vdots\\ 0\\ \mathbf{g}_n^{(K-1)}\\ 0\end{bmatrix} - \gamma^{-1}(n-1)\begin{bmatrix}0\\ \mathbf{g}_{n-1}^{(0)}\\ 0\\ \mathbf{g}_{n-1}^{(1)}\\ 0\\ \vdots\\ 0\\ \mathbf{g}_n^{(K-1)}\end{bmatrix}$$

$$\begin{aligned} &= \lambda^{-1}\begin{bmatrix}\Delta\mathbf{P}_{n-1}^{(0,0)} & \cdots & \Delta\mathbf{P}_{n-1}^{(0,K-1)}\\ \vdots & \ddots & \vdots\\ \Delta\mathbf{P}_{n-1}^{(0,K-1)} & \cdots & \Delta\mathbf{P}_{n-1}^{(K-1,K-1)}\end{bmatrix}\begin{bmatrix}\bar{h}(n,0)\\ \mathbf{c}_{n-1,0}\\ \vdots\\ \bar{h}(n,K-1)\\ \mathbf{c}_{n-1,K-1}\end{bmatrix} \\ &= \lambda^{-1}\Delta\mathbf{P}_{n-1}\begin{bmatrix}\bar{h}(n,0)\\ \mathbf{c}_{n-1,0}\\ \vdots\\ \bar{h}(n,K-1)\\ \mathbf{c}_{n-1,K-1}\end{bmatrix}\end{aligned} \tag{2.35}$$

where $\Delta\mathbf{P}_{n-1}$ is of dimension $K(N+1)\times K(N+1)$. Hence we have derived an update equation for the gain vector that depends only on the difference between the correlation matrix at time n and $n-1$, defined as $\Delta\mathbf{P}_{n-1}$. Now the update equation of conversion factor in terms of $\Delta\mathbf{P}_{n-1}$ is obtained by manipulating (2.26) as,

$$\gamma(n)^{-1} - \gamma(n-1)^{-1} = \lambda^{-1}\left[\mathbf{u}_n^H\mathbf{P}_{n-1}\mathbf{u}_n - \mathbf{u}_{n-1}^H\mathbf{P}_{n-2}\mathbf{u}_{n-1}\right] \tag{2.36}$$

The RHS of (2.36) can be expressed as,

$$\lambda^{-1}\left[\bar{h}(n,0)^*, \mathbf{c}_{n-1,0}^H, \cdots, \bar{h}(n,K-1)^*, \mathbf{c}_{n-1,K-1}^H\right] \Delta\mathbf{P}_{n-1} \begin{bmatrix} \bar{h}(n,0) \\ \mathbf{c}_{n-1,0} \\ \vdots \\ \bar{h}(n,K-1) \\ \mathbf{c}_{n-1,K-1} \end{bmatrix} \tag{2.37}$$

The regularization factor $\mathbf{\Pi}$ should be selected in a special way so that an efficient calculation of $\Delta\mathbf{P}_{n-1}$ is possible. This is because the initial value of $\Delta\mathbf{P}_{n-1}$ is defined as,

$$\begin{aligned} \Delta\mathbf{P}_{-1} &= \begin{bmatrix} \begin{bmatrix} \mathbf{P}_{-1}^{(0,0)} & \mathbf{0}_{N\times 1} \\ \mathbf{0}_{1\times N} & 0 \end{bmatrix} & \cdots & \begin{bmatrix} \mathbf{P}_{-1}^{(0,K-1)} & \mathbf{0}_{N\times 1} \\ \mathbf{0}_{1\times N} & 0 \end{bmatrix} \\ \vdots & \ddots & \vdots \\ \begin{bmatrix} \mathbf{P}_{-1}^{(K-1,0)} & \mathbf{0}_{N\times 1} \\ \mathbf{0}_{1\times N} & 0 \end{bmatrix} & \cdots & \begin{bmatrix} \mathbf{P}_{-1}^{(K-1,K-1)} & \mathbf{0}_{N\times 1} \\ \mathbf{0}_{1\times N} & 0 \end{bmatrix} \end{bmatrix} \\ &- \begin{bmatrix} \begin{bmatrix} 0 & \mathbf{0}_{1\times N} \\ \mathbf{0}_{N\times 1} & \mathbf{P}_{-2}^{(0,0)} \end{bmatrix} & \cdots & \begin{bmatrix} 0 & \mathbf{0}_{1\times N} \\ \mathbf{0}_{N\times 1} & \mathbf{P}_{-2}^{(0,K-1)} \end{bmatrix} \\ \vdots & \ddots & \vdots \\ \begin{bmatrix} 0 & \mathbf{0}_{1\times N} \\ \mathbf{0}_{N\times 1} & \mathbf{P}_{-2}^{(0,K-1)} \end{bmatrix} & \cdots & \begin{bmatrix} 0 & \mathbf{0}_{1\times N} \\ \mathbf{0}_{N\times 1} & \mathbf{P}_{-2}^{(K-1,K-1)} \end{bmatrix} \end{bmatrix} \end{aligned} \tag{2.38}$$

The inverse of $(l,k)^{th}$ block of $\mathbf{\Pi}$ is assigned as the value of the $(l,k)^{th}$ block of $\mathbf{P}_{-1}$. It is selected as,

$$\mathbf{P}_{-1}^{(l,k)} = \left[\mathbf{\Pi}^{(l,k)}\right]^{-1} = \frac{1}{\delta} \cdot diag\left\{\lambda^2, \lambda^3, \ldots, \lambda^{N+1}\right\} \tag{2.39}$$

and,

$$\mathbf{P}_{-2}^{(l,k)} = \lambda\left[\mathbf{\Pi}^{(l,k)}\right]^{-1} = \frac{1}{\delta} \cdot diag\left\{\lambda^3, \lambda^4, \ldots, \lambda^{N+2}\right\}, \tag{2.40}$$

where δ is the regularization parameter and is selected as [21],[22];

$$\delta = \sigma_{\mathbf{u}}^2(1-\lambda) \tag{2.41}$$

The variance of the input signal to the adaptive filter (2.8) is $\sigma_{\mathbf{u}}^2$ and λ is the forgetting factor.

Hence each $(l,k)^{th}$ block of $\Delta\mathbf{p}_{-1}$ is,

$$\Delta\mathbf{p}_{-1}^{(l,k)} = \frac{\lambda^2}{\delta} diag\left\{1, 0, \cdots, 0, -\lambda^N\right\} \tag{2.42}$$

Thus we observe that each $N \times N$ block of $\mathbf{P}_{-1}$ is reduced to a rank-2 matrix. It is proved in [15] that the rank-2 property of $\mathbf{P}_{-1}^{(l,k)}$ is invariant over time (*this is the case of single channel fast array RLS*). Hence we claim that this property holds for our scenario of FAM 2D-RLS. Factorizing $\Delta\mathbf{P}_{-1}$ as,

$$\Delta\mathbf{P}_{-1} = \lambda\bar{\mathbf{L}}_{-\mathbf{1}}\mathbf{S}_{-1}\bar{\mathbf{L}}_{-1}^H \tag{2.43}$$

where $\bar{\mathbf{L}}_{-1}$ of dimension $K(N+1) \times 2$ is defined as,

$$\bar{\mathbf{L}}_{-1} = \begin{bmatrix} \bar{\mathbf{L}}_{-1}^{(0)} \\ \bar{\mathbf{L}}_{-1}^{(1)} \\ \vdots \\ \bar{\mathbf{L}}_{-1}^{(K-1)} \end{bmatrix} \tag{2.44}$$

and

$$\bar{\mathbf{L}}_{-1}^{(k)} = \sqrt{\eta\lambda} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & \lambda^{N/2} \end{bmatrix} \tag{2.45}$$

is of dimension $(N+1) \times 2$. The signature matrix is defined as $\mathbf{S}_{-1} = diag\{1, -1\}$. Due to the rank preserving property of $\Delta\mathbf{P}_{n-1}$ the signature matrix remains a constant across time. Then at time n we can write,

$$\Delta\mathbf{P}_{n-1} = \bar{\mathbf{L}}_{n-1}\mathbf{S}\bar{\mathbf{L}}_{n-1}^H \tag{2.46}$$

Substituting (2.46) in (2.36),(2.37) and writing them in the array form we obtain the pre-array and post-array [24]. Defining $\mathbf{J} = diag\{1, \mathbf{S}\}$, the objective is to find a J-unitary matrix Θ_n that annihilates the last two elements in the first row of the pre-array(LHS of 2.47). This could be done by making use of Householder or Givens rotations [24],[29]. Then g_n is propagated iteratively as shown (2.47),[24].

$$
\begin{bmatrix}
\gamma^{-1/2}(n-1) & \left[\bar{h}(n,0)^*, \mathbf{c}_{n-1,0}^H \ldots \bar{h}(n,K-1)^* \mathbf{c}_{n-1,K-1}^H\right] \bar{\mathbf{L}}_{n-1} \\
\begin{bmatrix} 0 \\ \mathbf{g}_{n-1}^{(0)}\gamma^{-1/2}(n-1) \\ \vdots \\ 0 \\ \mathbf{g}_{n-1}^{(K-1)}\gamma^{-1/2}(n-1) \end{bmatrix} & \begin{bmatrix} \bar{\mathbf{L}}_{n-1}^{(0)} \\ \bar{\mathbf{L}}_{n-1}^{(1)} \\ \vdots \\ \bar{\mathbf{L}}_{n-1}^{(K-1)} \end{bmatrix}
\end{bmatrix} \Theta_n
$$

$$
= \begin{bmatrix}
\gamma^{-1/2}(n) & \mathbf{0}_{1\times 2} \\
\begin{bmatrix} \mathbf{g}_n^{(0)}\gamma^{-1/2}(n) \\ \vdots \\ 0 \\ \mathbf{g}_n^{(K-1)}\gamma^{-1/2}(n) \\ 0 \end{bmatrix} & \sqrt{\lambda} \begin{bmatrix} \bar{\mathbf{L}}_{n-1}^{(0)} \\ \bar{\mathbf{L}}_{n-1}^{(1)} \\ \vdots \\ \bar{\mathbf{L}}_{n-1}^{(K-1)} \end{bmatrix}
\end{bmatrix} \tag{2.47}
$$

The weight vector $\mathbf{W}_n$ is updated as,

$$
\mathbf{W}_n = \mathbf{W}_{n-1} + (\mathbf{h}_n - \mathbf{W}_n \mathbf{u}_n) \left[\mathbf{g}_n \gamma^{-1/2}(n)\right]^H \left[\gamma^{-1/2}(n)^{-1}\right] \tag{2.48}
$$

where $\mathbf{g}_n\gamma^{-1/2}(n)$ and $\gamma^{-1/2}(i)^{-1}$ are read from the post-array in (2.47).

2.4 Steady State Analysis of FAM 2D-RLS

Let the optimal weight matrix in the MMSE sense be $\mathbf{W}^o$. The aim of this section is to analyze the proximity of the weight matrix obtained by FAM 2D-RLS in steady state to that of $\mathbf{W}^o$. The analysis is based on the energy conservation method described in [24]. But unlike in [24], where the weight is a vector and the desired data is scalar, our scenario consist of a weight matrix and the desired data is a vector. Since $\mathbf{g}_n = \mathbf{P}_n\mathbf{u}_n$ we can rewrite (2.29) as,

$$
\mathbf{W}_n = \mathbf{W}_{n-1} + \mathbf{e}_n \mathbf{u}_n^H \mathbf{P}_n^H \tag{2.49}
$$

subtracting both sides of the above equation by $\mathbf{W}^o$,

$$
\widetilde{\mathbf{W}}_n = \widetilde{\mathbf{W}}_{n-1} - \mathbf{e}_n \mathbf{u}_n^H \mathbf{P}_n^H \tag{2.50}
$$

multiplying both sides by $\mathbf{u}_n$ from the right,

$$\mathbf{e}_n^{aps} = \mathbf{e}_n^{apr} - \mathbf{e}_n \|\mathbf{u}_n\|^2_{\mathbf{P}_n^H} \tag{2.51}$$

where $\mathbf{e}_n^{aps}$ is the *a posteriori* error vector , $\mathbf{e}_n^{apr}$ is the *a priori* error vector and $\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H} = \mathbf{u}_n^H \mathbf{P}_n^H \mathbf{u}_n$ is the weighted norm of $\mathbf{u}_n$. Substituting for $\mathbf{e}_n$ in (2.50) by making use of (2.51),

$$\widetilde{\mathbf{W}}_n + \mathbf{e}_n^{apr} \frac{\mathbf{u}_n^H \mathbf{P}_n^H}{\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}} = \widetilde{\mathbf{W}}_{n-1} + \mathbf{e}_n^{aps} \frac{\mathbf{u}_n^H \mathbf{P}_n^H}{\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}} \tag{2.52}$$

Taking Frobenius norm weighted by $(\mathbf{P}_n^H)^{-1}$ on both sides of (2.52),

$$\left\| \widetilde{\mathbf{W}}_n + \mathbf{e}_n^{apr} \frac{\mathbf{u}_n^H \mathbf{P}_n^H}{\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}} \right\|^2_{F,(\mathbf{P}_n^H)^{-1}} = \left\| \widetilde{\mathbf{W}}_{n-1} + \mathbf{e}_n^{aps} \frac{\mathbf{u}_n^H \mathbf{P}_n^H}{\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}} \right\|^2_{F,(\mathbf{P}_n^H)^{-1}} \tag{2.53}$$

where

$$\|\mathbf{A}\|^2_{F,\mathbf{K}} = tr\{\mathbf{A}\mathbf{K}\mathbf{A}^H\} \tag{2.54}$$

Expanding (2.53) by making use of (2.54), and cancelling out cross product terms by substituting $\mathbf{e}_n^{aps} = \mathbf{W}_n \mathbf{u}_n$ and $\mathbf{e}_n^{apr} = \mathbf{W}_{n-1} \mathbf{u}_n$ we obtain,

$$\|\widetilde{\mathbf{W}}_n\|^2_{F,(\mathbf{P}_n^H)^{-1}} + \frac{\|\mathbf{e}_n^{aps}\|^2}{\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}} = \|\widetilde{\mathbf{W}}_{n-1}\|^2_{F,(\mathbf{P}_n^H)^{-1}} + \frac{\|\mathbf{e}_n^{apr}\|^2}{\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}} \tag{2.55}$$

This equation is the energy conservation relation for the case of an RLS filter having a weight matrix. In order to obtain the variance relation [24] we have to find the average of the covariance matrix $\mathbf{P}_n^H$. This is defined in [24] as,

$$E[\mathbf{P}_n^H] = (1 - \lambda)\mathbf{R}_u^{-1} = \mathbf{P} \tag{2.56}$$

Since at steady state,

$$E[\|\widetilde{\mathbf{W}}_n\|^2_{F,(\mathbf{P}_n^H)^{-1}}] = E[\|\widetilde{\mathbf{W}}_{n-1}\|^2_{F,(\mathbf{P}_n^H)^{-1}}] \tag{2.57}$$

(2.55) can be reduced to,

$$E\left[\frac{\|\mathbf{e}_n^{aps}\|^2}{\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}}\right] = E\left[\frac{\|\mathbf{e}_n^{apr}\|^2}{\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}}\right] \quad n \to \infty \tag{2.58}$$

substituting for $\mathbf{e}_n^{aps}$ from (2.51) into (2.58) and expanding we can rewrite (2.58) as,

$$E\left[\frac{\|\mathbf{e}_n^{apr}\|^2 + \|\mathbf{e}_n\|^2\|\mathbf{u}_n\|^4_{\mathbf{P}_n^H} - 2Re\left\{\mathbf{e}_n^{apr,H}\mathbf{e}_n\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}\right\}}{\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}}\right] = E\left[\frac{\|\mathbf{e}_n^{apr}\|^2}{\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}}\right] \tag{2.59}$$

cancelling equal terms in (2.59), we obtain the variance relation [24] as,

$$E[\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}\|\mathbf{e}_n\|^2] = 2Re\{E[\mathbf{e}_n^{apr,H}\mathbf{e}_n]\} \tag{2.60}$$

Now assume a linear regressor model,

$$\mathbf{h}_n = \mathbf{W}^o\mathbf{u}_n + \mathbf{v}_n \tag{2.61}$$

where $\mathbf{v}_n$ is zero mean i.i.d random vector with covariance matrix $\mathbf{R}_v = \sigma_v^2\mathbf{I}_{K\times K}$. Also assume that $\mathbf{v}_i$ is independent of all $\mathbf{u}_j$ for all i, j and the initial weight matrix $\mathbf{W}_{-1}$ is independent of all $\{\mathbf{h}_n, \mathbf{v}_n, \mathbf{u}_n\}$. The above assumptions lead to the following relation [24],

$$\mathbf{e}_n = \mathbf{e}_n^{apr} + \mathbf{v}_n \tag{2.62}$$

Making use of (2.62), we can rewrite (2.60) as,

$$tr\{\mathbf{R}_v\}E[\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}] + E[\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}\|\mathbf{e}_n^{apr}\|^2] = 2E[\|\mathbf{e}_n^{apr}\|^2] = 2\xi^{RLS} \tag{2.63}$$

where ξ^{RLS} is the Excess Mean Square Error (EMSE). Making use of (2.56) and the separation property i.e assuming that at steady state, $\|\mathbf{u}_n\|^2_{\mathbf{P}_n^H}$ is independent of $\mathbf{e}_n^{apr}$ [24], an expression for EMSE is obtained from (2.63) as,

$$\xi^{RLS} = \frac{tr\{\mathbf{R}_v\}(1-\lambda)NK}{2-(1-\lambda)NK} \tag{2.64}$$

when $\lambda \approx 1$ (*which is normally the case*),

$$\xi^{RLS} = \frac{tr\{\mathbf{R}_v\}(1-\lambda)NK}{2} \tag{2.65}$$

Hence the Mean Square Error (MSE) at steady state is,

$$MSE = \xi^{RLS} + tr\{\mathbf{R}_v\} \tag{2.66}$$

Algorithm	$\times$	$+$
FAM 2D-RLS	$6NK + 10$	$10NK + 16$
2D-RLS	$(NK)^2 + 5NK + 2$	$(NK)^2 + 3NK$
2D-NLMS	$3NK + 2$	$3NK$

Table 2.1: Computational Complexity per Iteration in terms of complex multiplication and complex addition for estimating a single row of weight matrix.

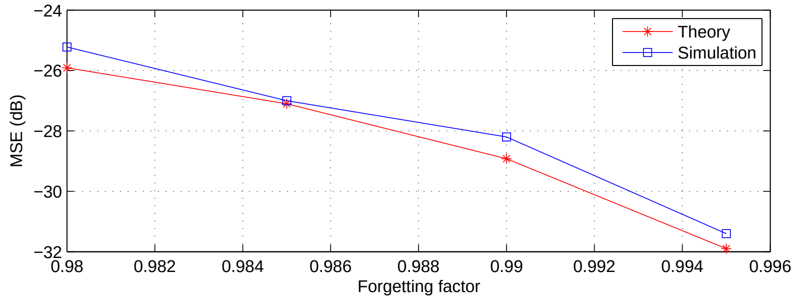

Figure 2.3: Theoretical and simulated MSE for FAM 2D-RLS

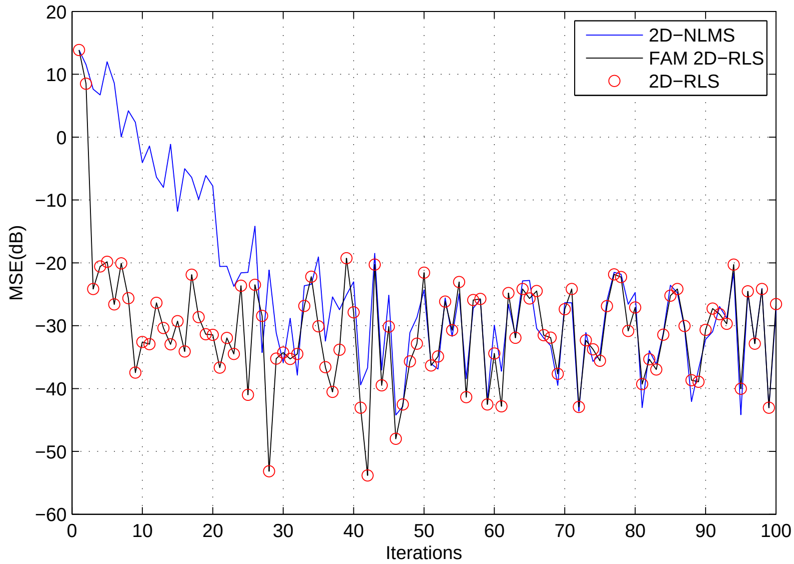

Figure 2.4: Learning curve for FAM 2D-RLS, 2D-NLMS & 2D-RLS

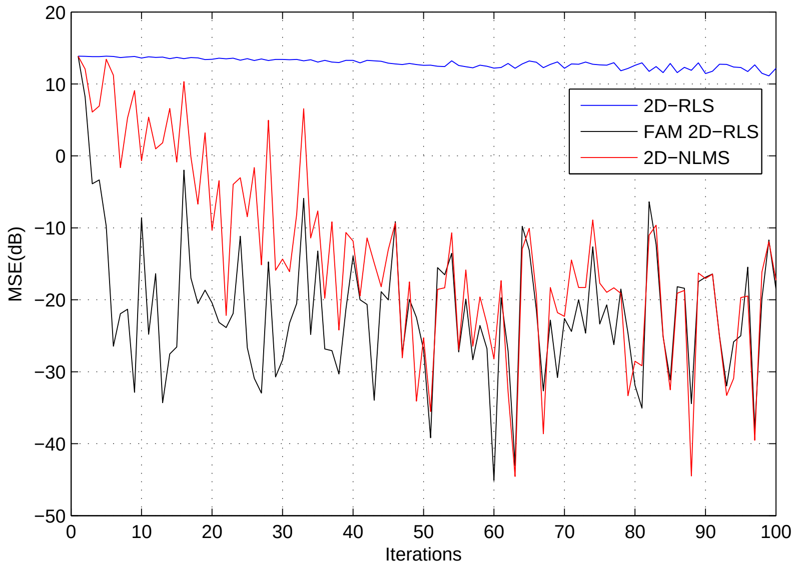

Figure 2.5: Numerical stability analysis of FAM 2D-RLS, 2D-NLMS & 2D-RLS for 16 bit quantized data

2.5 Simulation Setup

A SISO 256 point OFDM system is considered. The modulation technique is QPSK. A five path Rayleigh fading channel with tap delay line is considered and the CP length is taken as four so as to eliminate ISI. The carrier frequency is 2.5 GHz and channel bandwidth is 8 MHz. The device is assumed to move with a velocity of 60 km/hr.

2.6 Results and Analysis

The computational complexity of various 2D adaptive filters is shown in Table 2.1. The Fast array 2D-RLS algorithm has a computational complexity comparable to that of 2D-NLMS i.e. $O(NK)$. In Figure 2.3, the theoretical and simulated MSE is plotted for various values of forgetting factor assuming $\sigma_{\mathbf{u}}^2 = 1$. The optimal weight matrix $\mathbf{W}^o$ is a $K \times NK$ complex Gaussian matrix with zero mean and unit variance. In Figure 2.3, it is seen that $\lambda = 0.995$ gives the best MSE and hence is chosen as the value of forgetting factor for our simulations. The regularization parameter i.e. δ is obtained by substituting $\lambda = 0.995$ and $\sigma_v^2 = 0.001$ in (2.41). It is shown in Figure 2.4 that FAM 2D-RLS and 2D-RLS converges in about 8 iterations while it takes about 80 iterations for the 2D-NLMS to converge. In Figure 2.5 the numerical stability of FAM 2D-RLS is compared with 2D-RLS [13] and 2D-NLMS [31] by quantizing the input data vector and filter coefficients to 16 bits. It is observed that FAM 2D-RLS and NLMS converges using quantized data, while 2D-RLS does not converge. The better numerical stability of FAM

2D-RLS compared to 2D-RLS is because the former is implemented using the array method [24] while the latter is implemented using the classical algorithm of RLS [24].

2.7 Conclusion

In this chapter we proposed an adaptive OFDM channel estimation in the frequency domain that makes use of Fast array multichannel 2D-RLS technique. It was shown that the computational cost of the filter is comparable to that of 2D-NLMS while providing the same convergence property as 2D-RLS algorithm. Hence this low complexity filter could be used to track fast varying OFDM channels. Also it was shown that this filter is numerically stable compared to 2D-RLS filter. Thus we conclude that this channel estimation technique for OFDM improves upon the algorithms proposed in [13],[14],[31].

Chapter 3

FAM 2D-RLS Based Channel Estimation for Two-Way Relay Systems

3.1 Introduction

The field of cooperative communication is considered by many to be the future of wireless communication [49]. The basic idea of cooperative communication is that, dedicated terminals or user nodes called relays assist in conveying information between two communicating nodes. The use of relays help in increasing the link quality, reliability and data rate of the system. The initial relay systems proposed were one-way relay systems i.e any node S_2 will not be able to transmit data to S_1 at the same time as S_1 is transmitting data to S_2. So in the case of a two hop network two time slots are required for S_1 to convey its message to S_2 while it takes another two time slots for S_2 to convey its message to S_1. If the relay was not present, the whole communication process would have been over in two time slots. This decrease of spectral efficiency of relay systems is termed as resource devouring worms [50]. In order to alleviate this problem the concept of two-way relay system was introduced in [8]. In two-way relaying scheme, both S_1 and S_2 transmit data to the relay simultaneously. This is known as transmit phase. In the next time slot known as broadcast phase, the relay broadcasts this data after performing some signal processing on it. This relaying concept is similar to network coding [51]. Two-way relay systems is compatible with the Long Term Evolution (LTE) standard as discussed in [2],[3]. The advantage of bandwidth efficiency of two-way relay systems over one-way relay system comes at an extra cost of advanced signal processing to be performed at the relay or source nodes.

OFDM is a prominent modulation technique in latest communication standards like WIMAX and LTE [2] due to its ability to provide high data rates, robustness to intersymbol interference(ISI) and ease of implementation [32]. It was first combined with two-way relay system in [9] to obtain OFDM based two-way relay system. Some of the later works in this area are

[52], [53]. For coherent detection since the channel has to be known at the receiver [1], we propose a FAM 2D-RLS [15] based OFDM channel estimation for two-way relay system. Channel estimation scheme for two-way relay system with node capability i.e. relay systems that only performs a simple amplification of the received data while complex signal processing is performed by the nodes is proposed. This is the standard Amplify-Forward (AF) [9] scheme and in this thesis it is known as two-way relay with node capability. We further reduce the complexity of FAM 2D-RLS by introducing Block FAM 2D-RLS (BFAM 2D-RLS) [17]. The idea is to break up the channel frequency response vector into subvectors and each of the subvectors are estimated by parallel FAM 2D-RLS filters. A thorough simulation analysis of this scheme is performed. Two-way relay with node capability is more practical in a cooperative communication scenario than a two-way relay with relay capability which was discussed in Chapter-1. This is because in cooperative communication usually the relay consists of third party mobile devices which would be capable of carrying out complex signal processing tasks. But in order to have completeness we provide a FAM 2D-RLS based channel estimation scheme for relay capable two-way relay system.

3.2 Two-Way Relay With Node Capability

3.2.1 System Model

Let S_1 and S_2 be two source nodes and R be the relay node. All the devices consist of a single antenna. The system model is as shown in Figure 3.1. The communication process in two-way relay is divided into two phases. During the transmission phase, the source nodes convert the serial data symbols into K parallel data streams . Let the $K \times 1$ data vector at node $S_q, q = 1, 2$ be $\mathbf{x}^{(q)} = \left[x_0^{(q)}, \cdots, x_{K-1}^{(q)}\right]^T$. Then a K-point IFFT is performed to obtain the time domain data vector $\tilde{\mathbf{x}}^{(q)}, q = 1, 2$. If the length of the channel between S_1 and R is L_1 and that between S_2 and R is L_2 then a Channel Prefix (CP) of length $CP \geq max(L_1, L_2) - 1$ is added to the time domain data vector at each node. Then the data vectors are simultaneously send to R. After the removal of CP and performing a K-point DFT, the data received at R during the transmission phase is,

$$\mathbf{y}^{(R)} = \mathbf{X}^{(1)}\mathbf{c}^{(1)} + \mathbf{X}^{(2)}\mathbf{c}^{(2)} + \mathbf{n}^{(R)} \tag{3.1}$$

where $\mathbf{X}^{(q)} = diag\{\mathbf{x}^{(q)}\},\ q = 1, 2$. The channel frequency response vector between $S_1 - R$ and $S_2 - R$ is $\mathbf{c}^{(1)}, \mathbf{c}^{(2)}$ respectively . The maximum power available at the relay is $P^{(R)}$ while the maximum power at S_1 and S_2 is $P^{(1)}$ and $P^{(2)}$ respectively. The received data is multiplied by a diagonal matrix of dimension $K \times K$. Each of the diagonal element α_k is known as the

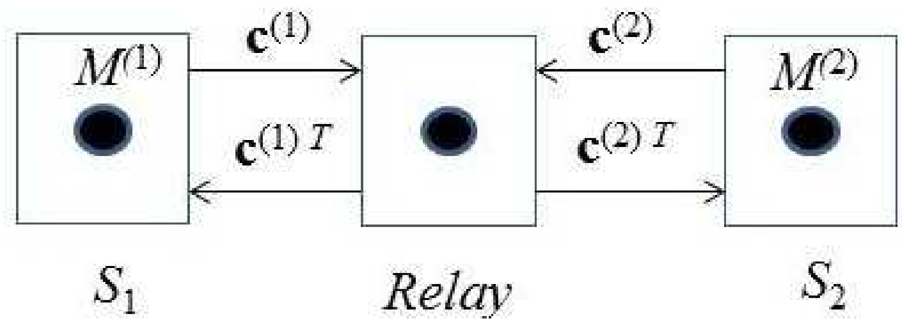

Figure 3.1: Two-way relay system with node capability

amplifying factor of subcarrier k and is defined as,

$$\alpha_k = \sqrt{\frac{P_k^{(R)}}{|c_k^{(1)}|^2 P_k^{(1)} + |c_k^{(2)}|^2 P_k^2 + N^{(R)}}} \tag{3.2}$$

where $P_k^{(1)}, P_k^{(2)}$ and $P_k^{(R)}$ is the power provided for subcarrier k at, S_1, S_2 and relay respectively. In this chapter the power at all the subcarriers are equally distributed. Hence α_k is a constant across subcarriers and is equal to α. During the broadcast phase, the K-Point DFT of $\mathbf{y}^{(R)}$ is performed and CP is added. Then this data vector is broadcasted. Due to symmetry, only the data received at S_1 is considered. Since the two-way relay system considered in this chapter is Time Division Duplex (TDD), the reverse channel can be assumed to be same as the forward channel. After the removal of CP and performing the K-point DFT of the data received at S_1 and S_2, it is given respectively as,

$$\mathbf{y}^{(1)} = \alpha(\mathbf{X}^{(1)}\mathbf{c}^{(1)} \odot \mathbf{c}^{(1)} + \mathbf{X}^{(2)}\mathbf{c}^{(2)} \odot \mathbf{c}^{(1)} + \mathbf{c}^{(1)} \odot \mathbf{n}^{(R)}) + \mathbf{n}^{(1)} \tag{3.3}$$

and

$$\mathbf{y}^{(2)} = \alpha(\mathbf{X}^{(2)}\mathbf{c}^{(2)} \odot \mathbf{c}^{(2)} + \mathbf{X}^{(1)}\mathbf{c}^{(2)} \odot \mathbf{c}^{(1)} + \mathbf{c}^{(2)} \odot \mathbf{n}^{(R)}) + \mathbf{n}^{(2)} \tag{3.4}$$

The first term in (3.3),(3.4) is known as self interference, because it consists of the actual data $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$ that was transmitted by S_1, S_2 respectively. So if $\mathbf{c}^{(1)} \odot \mathbf{c}^{(1)}$ is known at S_1 and $\mathbf{c}^{(2)} \odot \mathbf{c}^{(2)}$ is known at S_2, then the self interference could be removed by subtracting the first terms from the received data. After the self interference removal , the knowledge of $\mathbf{c}^{(2)} \odot \mathbf{c}^{(1)}$ could be used to detect $\mathbf{X}^{(1)}$ at S_2 and $\mathbf{X}^{(2)}$ at S_1. Since (3.3),(3.4) have the same structure, the channel estimation algorithm implemented at S_1 and S_2 is the same. Hence from now on, we consider channel estimation at S_1.

Let $\mathbf{h}^{(1)} = \mathbf{c}^{(1)} \odot \mathbf{c}^{(1)}$ and $\mathbf{g}^{(1)} = \mathbf{c}^{(1)} \odot \mathbf{c}^{(2)}$. The noise term at S_1 is $\mathbf{w}^{(1)} = \alpha\mathbf{c}^{(1)} \odot \mathbf{n}^{(R)} + \mathbf{n}^{(1)}$. Hence,

$$\mathbf{y}^{(1)} = \alpha(\mathbf{X}^{(1)}\mathbf{h}^{(1)} + \mathbf{X}^{(2)}\mathbf{g}^{(1)}) + \mathbf{w}^{(1)} \tag{3.5}$$

This equation states that a single antenna based two-way relay system with node capability can be considered as a Multiple Input Single Output (MISO) system. Hence the channel estimation technique implemented in [57] could be used to find the LS estimate of the channels $\mathbf{h}^{(1)}$ and $\mathbf{g}^{(1)}$. Another important observation which is made in (3.5) is that unlike the non relay based communication system, in a two-way relay with node capability we have to estimate two channel coefficient vectors $\mathbf{h}^{(1)}$ and $\mathbf{g}^{(1)}$. These channel coefficients in turn consists of the $\mathbf{c}^{(1)}$ and $\mathbf{c}^{(2)}$. Hence we call $\mathbf{h}^{(1)}$ and $\mathbf{g}^{(1)}$ as the combined channel coefficients while $\mathbf{c}^{(1)}$ and $\mathbf{c}^{(2)}$ is known as the individual channels.

3.2.2 Least Square Channel Estimation

The Least Square (LS) cost function for estimating the channel coefficients is,

$$J(\bar{\mathbf{h}}^{(1)}, \bar{\mathbf{g}}^{(1)}) = (\mathbf{y}^{(1)} - \bar{\mathbf{y}}^{(1)})^H (\mathbf{y}^{(1)} - \bar{\mathbf{y}}^{(1)}) \tag{3.6}$$

where $\bar{\mathbf{y}}^{(1)} = \mathbf{X}^{(1)}\bar{\mathbf{h}}^{(1)} + \mathbf{X}^{(2)}\bar{\mathbf{g}}^{(1)}$ and $\bar{\mathbf{h}}^{(1)}$ and $\bar{\mathbf{g}}^{(1)}$ are the LS estimate of the channel $\mathbf{h}^{(1)}$ and $\mathbf{g}^{(1)}$ respectively. The LS estimate of the channel is obtained by finding the complex gradient of (3.6) w.r.t $\bar{\mathbf{h}}^{(1)H}$ and $\bar{\mathbf{g}}^{(1)H}$ and equating to zero. The gradient vectors are [58],

$$\nabla_{\bar{\mathbf{h}}^{(1)H}} = -\mathbf{X}^{(1)H}\mathbf{y}^{(1)} + \mathbf{X}^{(1)H}\mathbf{X}^{(1)}\bar{\mathbf{h}}^{(1)} + \mathbf{X}^{(1)H}\mathbf{X}^{(2)}\bar{\mathbf{g}}^{(1)} = 0 \tag{3.7}$$

$$\nabla_{\bar{\mathbf{g}}^{(1)H}} = -\mathbf{X}^{(2)H}\mathbf{y}^{(1)} + \mathbf{X}^{(2)H}\mathbf{X}^{(1)}\bar{\mathbf{h}}^{(1)} + \mathbf{X}^{(2)H}\mathbf{X}^{(2)}\bar{\mathbf{g}}^{(1)} = 0 \tag{3.8}$$

The above equations can be solved by assuming that the two source nodes send training data at orthogonal frequency locations. Let S_1 send preamble data at even frequency locations while no data is sent at odd frequency locations, and S_2 send preamble data at odd frequency locations while no data is sent at even frequency locations. Let $\mathbf{X}_p^{(1)}$ and $\mathbf{X}_p^{(2)}$ be the preamble symbol matrix send from S_1 and S_2 respectively. Let $\mathbf{X}_{p,even}^{(1)}$ and $\mathbf{X}_{p,odd}^{(2)}$ be the vector consisting of the non zero elements of $\mathbf{X}_p^{(1)}$ and $\mathbf{X}_p^{(2)}$ respectively. The LS estimate of channel frequency response at even and odd sample locations of $\mathbf{h}^{(1)}, \mathbf{g}^{(1)}$ using these preamble matrices are,

$$\bar{\mathbf{h}}_{even}^{(1)} = (\mathbf{X}_{p,even}^{(1)H}\mathbf{X}_{p,even}^{(1)})^{-1}(\mathbf{X}_{p,even}^{(1)H}\mathbf{y}^{(1)}) \tag{3.9}$$

$$\bar{\mathbf{g}}_{odd}^{(1)} = (\mathbf{X}_{p,odd}^{(2)H}\mathbf{X}_{p,odd}^{(2)})^{-1}(\mathbf{X}_{p,odd}^{(2)H}\mathbf{y}^{(2)}) \tag{3.10}$$

A simple interpolation technique could be used to obtain the full LS channel estimate $\bar{\mathbf{h}}^{(1)}$ and $\bar{\mathbf{g}}^{(1)}$. The linear interpolation for obtaining $\bar{\mathbf{h}}^{(1)}$ is,

$$\begin{aligned}
\bar{h}^{(1)}(k) &= \tfrac{1}{2}\left[\bar{h}^{(1)}_{even}(\tfrac{k-1}{2}) + \bar{h}^{(1)}_{even}\tfrac{k+1}{2}\right], \quad k \text{ is odd and } k \neq K-1. \\
\bar{h}^{(1)}(k) &= \bar{h}^{(1)}_{even}(\tfrac{k}{2}-1), \quad k \text{ is even.} \\
\bar{h}^{(1)}(K-1) &= \bar{h}^{(1)}_{even}(\tfrac{K}{2}-1).
\end{aligned} \tag{3.11}$$

The linear interpolation for obtaining $\bar{\mathbf{g}}^{(1)}$ is,

$$\begin{aligned}
\bar{g}^{(1)}(k) &= \tfrac{1}{2}\left[\bar{g}^{(1)}_{odd}(\tfrac{k}{2}) + \bar{g}^{(1)}_{odd}(\tfrac{k}{2}+1)\right], \quad k \text{ is even and } k \neq 0. \\
\bar{g}^{(1)}(k) &= \bar{g}^{(1)}_{odd}(\tfrac{k-1}{2}), \quad \text{k is odd.} \\
\bar{g}^{(1)}(0) &= \bar{g}^{(1)}_{odd}(0).
\end{aligned} \tag{3.12}$$

Since in the next section an adaptive filter is implemented, we include a subscript n to the LS channel frequency response to denote the estimate at time n. Thus the structure of the instantaneous LS channel estimate is,

$$\begin{aligned}
\bar{\mathbf{h}}^{(1)}_n &= \left[\bar{h}(n,0)^*, \cdots, \bar{h}(n,K-1)^*\right]^H \\
\bar{\mathbf{g}}^{(1)}_n &= \left[\bar{g}(n,0)^*, \cdots, \bar{g}(n,K-1)^*\right]^H
\end{aligned} \tag{3.13}$$

3.2.3 DDCE-OFDM

The implementation of DDCE-OFDM [16] even in the case of a single antenna two-way relay system is different from that of a SISO system. This is due to the presence of self interference. The self interference at S_1 is $\alpha\mathbf{X}^{(1)}_n\mathbf{h}^{(1)}_n$. Since $\mathbf{X}^{(1)}_n$ is known at S_1, the self interference component could be eliminated if an estimate of $\mathbf{h}^{(1)}_n$ is available. Assume that α and the refined estimate of the channel coefficients, $\hat{\mathbf{h}}^{(1)}_{n-1}$ and $\hat{\mathbf{h}}^{(2)}_{n-1}$ is available at S_1. The channel coefficients is assumed to remain constant for two symbol duration. Then at time n, DD estimator for single antenna OFDM two-way relay system functions as follows,
Step-1: Self interference cancellation,

$$\mathbf{s}^{(2)}_n = \mathbf{y}^{(1)}_n - \alpha\mathbf{X}^{(1)}_n\hat{\mathbf{h}}^{(1)}_{n-1} \tag{3.14}$$

Step-2: The LS estimate of $\mathbf{x}_n^{(2)}$ is obtained as,

$$\bar{\mathbf{x}}_n^{(2)} = (\alpha \hat{\mathbf{G}}_{n-1}^{(1)})^{-1} \mathbf{s}_n^{(2)} \tag{3.15}$$

where $\hat{\mathbf{G}}_n^{(1)} = diag\{\hat{\mathbf{g}}_n^{(1)}\}$.
Step-3: The estimated data $\mathbf{x}_n^{(2)}$ is sent to a detector
Step-4: Assuming that there is no error in detection, the LS estimate of $\mathbf{g}_n^{(1)}$ is obtained as,

$$\bar{\mathbf{g}}_n^{(1)} = (\alpha \mathbf{X}_n^{(2)})^{-1} \mathbf{s}_n^{(2)} \tag{3.16}$$

Step-5: $\bar{\mathbf{g}}_n^{(1)}$ is input to the adaptive filter to obtain a refined estimate $\hat{\mathbf{g}}_n^{(1)}$.
Step-6: The interference due to $\mathbf{x}_n^{(2)}$ is cancelled,

$$\mathbf{s}_n^{(1)} = \mathbf{y}_n^{(1)} - \alpha \mathbf{X}_n^{(2)} \hat{\mathbf{g}}_n^{(1)} \tag{3.17}$$

Step-7: The LS estimate of channel $\hat{\mathbf{h}}_n^{(1)}$ is obtained as,

$$\bar{\mathbf{h}}_n^{(1)} = (\alpha \mathbf{X}_n^{(1)})^{-1} \mathbf{s}_n^{(1)} \tag{3.18}$$

Step-8: $\bar{\mathbf{h}}_n^{(1)}$ is input to the adaptive filter to obtain a refined estimate $\hat{\mathbf{h}}_n^{(1)}$.
Step-9: Repeat steps 1 to 8 until OFDM symbols are send from S_1 and S_2.

3.3 Two-Way Relay with Relay Capability

3.3.1 System Model

The two-way relay system in this section consists of two nodes S_1 and S_2 that communicate with each other through the relay R, as shown in Figure 3.2. There is no direct path between S_1 and S_2. The nodes are considered to be single antenna devices. Unlike in the previous section, here the relay consists of at least two antennas. The reason for this will be evident later in this section. The communication process is divided into two phases, transmission phase and broadcast phase. During the transmission phase, the source nodes converts the serial data symbols into K parallel data streams . Let the $K \times 1$ data vector at node $S_q, q = 1, 2$ be $\mathbf{x}^{(q)}$. Then a K-point IFFT is performed to obtain the time domain data vector $\tilde{\mathbf{x}}^{(q)}, q = 1, 2$. If the length of the channel between S_1 and R is L_1 and that between S_2 and R is L_2 then a Channel Prefix (CP) of length $CP \geq max(L_1, L_2) - 1$ is added to the time domain data vector at each node. Then the data vectors are simultaneously sent to R. After the removal of CP and performing a K-point DFT, the data received at j^{th} antenna of R during the n^{th} transmission phase for a two antenna relay system is,

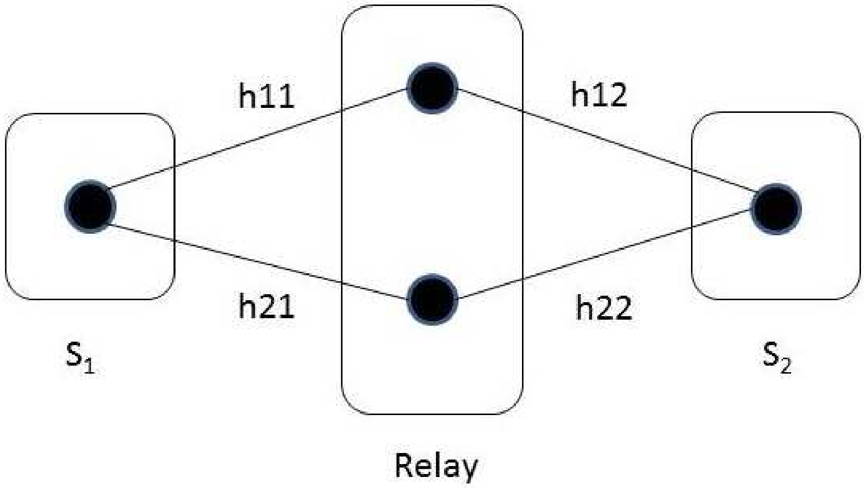

Figure 3.2: Two-way relay system with relay capability

$$\mathbf{y}_{n,j}^{(R)} = \mathbf{X}_n^{(1)}\mathbf{h}_n^{(j,1)} + \mathbf{X}_n^{(2)}\mathbf{h}_n^{(j,2)} + \mathbf{w}_n^{(j,R)},\ j = 1, 2 \tag{3.19}$$

where $\mathbf{X}_n^{(q)} = diag\{\mathbf{x}_n^{(q)}\},\ q = 1, 2$. The channel frequency response between node S_q and j^{th} antenna of R is $\mathbf{h}_n^{(j,q)},\ j, q = 1, 2$ and Additive White Gaussian Noise (AWGN) at antenna j of the relay is $\mathbf{w}_n^{(j,R)}$. The data received at the antennas can be combined and written in vector form as,

$$\mathbf{y}_n^{(R)} = \mathbf{H}_n\mathbf{x}_n + \mathbf{w}_n^{(R)} \tag{3.20}$$

where $\mathbf{x}_n = \left[\mathbf{x}_n^{(1)}, \mathbf{x}_n^{(2)}\right]^T$, $\mathbf{w}_n = \left[\mathbf{w}_{n,1}^{(R)}, \mathbf{w}_{n,2}^{(R)}\right]^T$ and,

$$\mathbf{H}_n = \begin{bmatrix} diag\{\mathbf{h}_n^{(1,1)}\} & diag\{\mathbf{h}_n^{(1,2)}\} \\ diag\{\mathbf{h}_n^{(2,1)}\} & diag\{\mathbf{h}_n^{(2,2)}\} \end{bmatrix} \tag{3.21}$$

It can be observed from (3.19),(3.20) that even a simple single antenna node based two-way relay system with relay processing capability requires multiple antennas at the relay in order to separate the signals from S_1 and S_2. This requires the channel frequency response to be known at the relay. It is also observed that the LS channel estimation at the relay requires the method implemented in [58] for a 2×2 MIMO system. Thus consider that S_1 send pilot symbols at even frequency samples while S_2 send pilot symbols at odd frequency samples.

3.3.2 Least Square Channel Estimation

Let's consider the first antenna at the relay. The data received is,

$$\mathbf{y}_{n,1}^{(R)} = \mathbf{X}_n^{(1)}\mathbf{h}_n^{(1,1)} + \mathbf{X}_n^{(2)}\mathbf{h}_n^{(1,2)} + \mathbf{w}_n^{(1,R)} \tag{3.22}$$

The aim is to find the LS estimate $\mathbf{h}^{(1,1)}$ and $\mathbf{h}^{(1,2)}$, which is denoted as $\bar{\mathbf{h}}^{(1,1)}$ and $\bar{\mathbf{h}}^{(1,2)}$ respectively. The cost function that has to be minimized is,

$$J\left(\bar{\mathbf{h}}^{(1,1)}, \bar{\mathbf{h}}^{(1,2)}\right) = \left(\mathbf{y}_n^{(1,R)} - \bar{\mathbf{y}}_{n,1}^{(R)}\right)^H \left(\mathbf{y}_n^{(1,R)} - \bar{\mathbf{y}}_n^{(1,R)}\right) \tag{3.23}$$

where $\bar{\mathbf{y}}_n^{(1,R)} = \mathbf{X}_n^{(1)}\bar{\mathbf{h}}_n^{(1,1)} + \mathbf{X}_n^{(2)}\bar{\mathbf{h}}_n^{(1,2)}$. Similar to the LS channel estimation technique to two-way relay with node capability, S_1 sends non-zero preamble data at even frequency locations while S_2 sends at odd frequency locations. The overall estimate of the channels can be obtained using a simple frequency interpolation technique as explained in (3.11),(3.12). The second antenna also performs similar LS estimation technique to obtain $\bar{\mathbf{h}}_n^{(2,1)}$ and $\bar{\mathbf{h}}_n^{(2,2)}$. After obtaining the initial LS estimate of the channels, in order to reduce the number of preamble symbols a DDCE based technique in implemented.

3.3.3 DDCE-OFDM

Let $\bar{\mathbf{H}}_n$ be the LS channel estimate of $\mathbf{H}_n$. In order to find a better estimate, the LS estimate of the channel is passed through an adaptive filter to obtain $\hat{\mathbf{H}}_n$. Assuming that the channel frequency response is approximately constant for two time slots, we can use the channel estimate at time $n-1$ to obtain the estimate of the transmitted data at time n. After passing this estimate through a decision device, the data sent from S_1 and S_2 can be used to find the estimate of the channel at time n. Due to the multiple antennas present at the relay, we have to implement a MIMO OFDM based DDCE. The steps required in obtaining the LS channel estimate using MIMO-DDCE is as follows,

Step-1: Using preambles, the LS estimate of $\mathbf{x}_n$ is obtained as,

$$\hat{\mathbf{H}}_{n-1}^{-1}\mathbf{y}_n^{(R)} = \mathbf{x}_n^{(R)} + \hat{\mathbf{H}}_{n-1}^{-1}\mathbf{w}_n^{(R)} \tag{3.24}$$

Step-2: LS estimates of data is send to detector to obtain $\mathbf{x}_n$.

Step-3: Subtracting the interference term due to $\mathbf{x}_n^{(1)}$ from (3.24),

$$\mathbf{s}_n^{(2)} = \mathbf{y}_n^{(R)} - \hat{\mathbf{H}}_{n-1}(:, 1:K)\mathbf{x}_n^{(1)} \tag{3.25}$$

where $\mathbf{H}_n(:, p : q)$ is the matrix containing the p^{th} to q^{th} column of $\mathbf{H}_n$.
Step-4: Obtain the LS estimate of $\mathbf{H}_n(:, K+1 : 2K)$,

$$\begin{aligned}(\mathbf{X}_n^{(2)})^{-1}\mathbf{s}_n^{(2)}(1:K) &= \mathbf{H}_n(1:K, K+1:2K) + (\mathbf{X}_n^{(2)})^{-1}\mathbf{w}_n^{(R)}(1:K) \\ (\mathbf{X}_n^{(2)})^{-1}\mathbf{s}_n^{(2)}(K+1:2K) &= \mathbf{H}_n(k+1:2K, K+1:2K) + (\mathbf{X}_n^{(2)})^{-1}\mathbf{w}_n^{(R)}(K+1:2K)\end{aligned} \tag{3.26}$$

Step-5: This LS estimate of partial channel frequency response is bettered using an adaptive filter to obtain $\hat{\mathbf{H}}_n(:, K+1 : 2K)$.
Step-6: The interference due to $\mathbf{x}_n^{(2)}$ is subtracted from (3.24) as,

$$\mathbf{s}_n^{(1)} = \mathbf{y}_n - \hat{\mathbf{H}}_n(:, K+1 : 2K)\mathbf{x}_n^{(2)} \tag{3.27}$$

Step-7: Obtain the LS estimate of $\mathbf{H}_n(:, 1 : K)$ as,

$$\begin{aligned}(\mathbf{X}_n^{(1)})^{-1}\mathbf{s}_n^{(1)}(1:K) &= \mathbf{H}_n(1:K, 1:K) + (\mathbf{X}_n^{(1)})^{-1}\mathbf{w}_n^{(R)}(1:K) \\ (\mathbf{X}_n^{(1)})^{-1}\mathbf{s}_n^{(1)}(K+1:2K) &= \mathbf{H}_n(K+1:2K, 1:K) + (\mathbf{X}_n^{(1)})^{-1}\mathbf{w}_n^{(R)}(K+1:2K)\end{aligned} \tag{3.28}$$

Step-8: This LS estimate of partial channel frequency response is bettered using an adaptive filter to obtain $\hat{\mathbf{H}}_n(:, 1 : K)$.
Step-9: Until S_1 and S_2 stops sending data, step 1-8 should be repeated.

3.3.4 Transceiver Matrix

In Chapter-1 it was assumed that the actual CSI was available at the relay. But in a practical scenario, it has to be estimated as explained previously. After the channel estimate $\hat{\mathbf{H}}_n$ is obtained at the relay, transceiver matrix could be designed as explained in [54],[55] and [56]. The advantage of having CSI at the relay is that it reduces the computational complexity at the nodes. But this method is feasible only in cases where the relay node is a dedicated system having enough power to perform complex signal processing tasks.

3.4 BFAM 2D-RLS Based Channel Estimation

In this section we first explain a general scenario of estimating a channel frequency response vector $\mathbf{h}_n$ of dimension $K \times 1$. Let the LS estimate of $\mathbf{h}_n$ be $\bar{\mathbf{h}}_n$ and the output of the adaptive filter be $\hat{\mathbf{h}}_n$. The adaptive channel estimation schemes for two-way relay with node and relay capability can be easily obtained by changing the variable $\mathbf{h}_n, \bar{\mathbf{h}}_n$ and $\hat{\mathbf{h}}_n$. Assuming that the channel is correlated for N OFDM symbols, define a vector $\bar{\mathbf{u}}_n$ of dimension $NK \times 1$ that contain LS channel estimate of past N time samples,

$$\mathbf{u}_n = \left[\mathbf{c}_{n,0}^H \cdots \mathbf{c}_{n,K-1}^H\right]^H \tag{3.29}$$

where

$$\mathbf{c}_{n,k} = \left[\bar{h}(n,k), \bar{h}(n-1,k), \cdots, \bar{h}(n-N+1,k)\right]^T \tag{3.30}$$

In the previous chapter we designed a filter called FAM 2D-RLS. It was assumed that each of the elements in the $K \times NK$ input vector to the adaptive filter was correlated among each other. If $\mathbf{u}_n$ is the input vector then the estimate is obtained as $\hat{\mathbf{h}}_n = \mathbf{W}_n \mathbf{u}_n$. This requires a computational complexity of $O(NK^2)$. In order to alleviate this problem, the assumption of correlation among all the channel frequency response samples can be relaxed. The $K \times 1$ channel frequency response can be considered to be made up of subvectors of dimension $\frac{K}{M} \times 1$ i.e. the channel frequency response is broken up into M blocks. Then M parallel adaptive filters with tap length of $\frac{K}{M} \times \frac{NK}{M}$ is used to estimate the channel frequency response. The advantage of this technique is that the computational complexity is reduced from $O(NK^2)$ to $O(\frac{NK^2}{M})$. But the drawback of this technique is that we consider only the correlation of frequency samples in each block. This will not give the full advantage of the 2D adaptive filter. This adaptive filtering scheme is called Block Fast Array Multichannel 2D-RLS (BFAM 2D-RLS). This filter consists of M parallel FAM 2D-RLS filters. The working of FAM 2D-RLS for estimating the m^{th} , $m = 1 \cdots M$ block is as follows.

The $\frac{NK}{M} \times 1$ input vector of each of the M adaptive filters is,

$$\mathbf{u}_n^{(m)} = \left[\mathbf{c}_{n,\frac{(m-1)K}{M}}^H \cdots \mathbf{c}_{n,\frac{mK}{M}-1}^H\right]^H, \; m = 1 \cdots M \tag{3.31}$$

and the estimate of m^{th} group of the channel frequency response is,

$$\hat{\mathbf{h}}_n^{(m)} = \mathbf{W}_n^{(m)} \mathbf{u}_n^{(m)}, \; m = 1 \cdots M \tag{3.32}$$

Assuming that $\mathbf{h}_n^{(m)}$ is the actual channel frequency response of m^{th} group at time n, the optimization problem solved by the BFAM 2D-RLS adaptive filter is,

$$\underset{\mathbf{W}_n^{(m)}}{min} \left[\lambda^{n+1} \mathbf{W}_n^{(m)} \mathbf{\Pi} \mathbf{W}_n^{(m)H} + \sum_{i=0}^{n} \lambda^{n-i} \|\mathbf{h}_n^{(m)} - \hat{\mathbf{h}}_n^{(m)}\|^2\right] \tag{3.33}$$

The regularization factor is $\mathbf{\Pi} = \delta \mathbf{I}_{\frac{NK}{M} \times \frac{NK}{M}}$. The forgetting factor is λ and the regularization parameter is δ.

As mentioned before, the aim is to design an adaptive filter that makes use of the time shift structure of the input data vector so as to reduce the computational complexity. In our case the input data vector is $\mathbf{u}_n^{(m)}$ (3.31). But it doesn't have a shift structure in the real sense. But observing (3.30), it is seen that $\mathbf{c}_{n,k}$ has time shift structure. Hence $\mathbf{u}_n^{(m)}$ has a block shift structure. Thus a $\frac{K}{M}$-channel adaptive filter could be designed with the input vector at the $k^{(th)}$ channel being $\mathbf{c}_{n,k}^{(m)}$ $k = 0 \cdots \frac{K}{M} - 1$.

Algorithm

Consider data $\left\{\mathbf{u}_j^{(m)}, \mathbf{h}_j^{(m)}\right\}_{j=0}^{n}$. The forgetting factor is λ and γ is the conversion factor. The gain vector of dimension $\frac{NK}{M} \times 1$ at time n is $\mathbf{g}_n$ and is partitioned as,

$$\mathbf{g}_n = \left[\mathbf{g}_n^{(0)} \; \mathbf{g}_n^{(1)} \; \ldots \; \mathbf{g}_n^{(\frac{K}{M}-1)}\right]^T \tag{3.34}$$

Let the inverse of the $\frac{NK}{M} \times \frac{NK}{M}$ regularization matrix be $\mathbf{\Pi}^{-1}$. Each of its (k,l) block is represented as,

$$\left[\mathbf{\Pi}^{(k,l)}\right]^{-1} = \frac{1}{\delta} \cdot diag\left\{\lambda^2, \lambda^3, \ldots, \lambda^{N+1}\right\}, \;\; \delta < 0. \tag{3.35}$$

Define a signature matrix $\mathbf{S} = diag\{1, -1\}$ and a matrix of dimension 3×3 as $\mathbf{J} = diag\{1, S\}$. The FAM 2D-RLS finds the solution $\mathbf{W}_n^{(m)}$ for the optimization problem in (3.33) in a recursive manner,

Step-1: Initialize,
$\mathbf{W}_{-1}^{(m)} = \mathbf{0}_{\frac{K}{M} \times \frac{NK}{M}}$, $\gamma^{-1/2}(-1) = 1$, $\mathbf{g}_{-1} = \mathbf{0}_{\frac{NK}{M} \times 1}$

Define a matrix of dimension $\frac{K}{M}(N+1) \times 2$ as,

$$\bar{\mathbf{L}}_{-1} = \begin{bmatrix} \bar{\mathbf{L}}_{-1}^{(0)} \\ \bar{\mathbf{L}}_{-1}^{(1)} \\ \vdots \\ \bar{\mathbf{L}}_{-1}^{(\frac{K}{M}-1)} \end{bmatrix} \tag{3.36}$$

where

$$\bar{\mathbf{L}}_{-1}^{(k)} = \sqrt{\eta\lambda} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & \lambda^{K/2M} \end{bmatrix} \tag{3.37}$$

is of dimension $(N+1) \times 2$. For $n \geq 0$, repeat the following steps,

Step-2: Find a J-unitary matrix Θ_n that annihilates the last two elements in the first row of the

Algorithm	$\times$	$+$
BFAM 2D-RLS	$\frac{6NK^2}{M}+10$	$\frac{10NK^2}{M}+16$
2D-RLS	$N^2K^3+5NK^2+2$	$N^2K^3+3NK^2$
2D-NLMS	$3NK^2+2$	$3NK^2$

Table 3.1: Computational Complexity of BFAM 2D-RLS per Iteration in terms of complex multiplication and complex addition

pre-array(LHS of (19)). Then $\mathbf{g}_n$ is propagated iteratively [18],[21] as,

$$\begin{bmatrix} \gamma^{-1/2}(n-1) & \left[u^m(n,0)\,\mathbf{u}^m_{n-1,0}\dots\; u^m(n,K-1)\mathbf{u}^m_{n-1,K-1}\right] \\ \begin{bmatrix} 0 \\ \mathbf{g}^{(0)}_{n-1}\gamma^{-1/2}(n-1) \\ \vdots \\ 0 \\ \mathbf{g}^{(K-1)}_{n-1}\gamma^{-1/2}(n-1) \end{bmatrix} & \begin{bmatrix} \bar{\mathbf{L}}^{(0)}_{n-1} \\ \bar{\mathbf{L}}^{(1)}_{n-1} \\ \vdots \\ \bar{\mathbf{L}}^{(K-1)}_{n-1} \end{bmatrix} \end{bmatrix} \Theta_n$$

$$= \begin{bmatrix} \gamma^{-1/2}(n) & \mathbf{0}_{1\times 2} \\ \begin{bmatrix} \mathbf{g}^{(0)}_{n}\gamma^{-1/2}(n) \\ \vdots \\ 0 \\ \mathbf{g}^{(K-1)}_{n}\gamma^{-1/2}(n) \\ 0 \end{bmatrix} & \sqrt{\lambda}.\begin{bmatrix} \bar{\mathbf{L}}^{(0)}_{n} \\ \bar{\mathbf{L}}^{(1)}_{n} \\ \vdots \\ \bar{\mathbf{L}}^{(K-1)}_{n} \end{bmatrix} \end{bmatrix} \tag{3.38}$$

Step-3: The weight vector $\mathbf{W}^{(q)}_n$ is updated as

$$\mathbf{W}^{(m)}_n = \mathbf{W}^{(m)}_{n-1} + \left(\mathbf{h}^{(m)}_n - \mathbf{W}^{(m)}_n\mathbf{u}^{(m)}_n\right)\left[\mathbf{g}_n\gamma^{-1/2}(n)\right]^H \gamma^{1/2}(n) \tag{3.39}$$

where $\mathbf{g}_n\gamma^{-1/2}(n)$ and $\gamma^{-1/2}(i)^{-1}$ are read from the post-array in (25).

After the convergence of the filter, the transmitter starts sending data symbols. During this phase the LS estimate of the channel is obtained using the Decision Directed(DD) [16] technique discussed in the previous section .

3.5 Simulation setup

The computer simulation is performed in MATLAB for 16-QAM OFDM based two-way relay systems. The number of subchannel is 64. The pedestrian-A (pedA), pedestrian-B (pedB) and

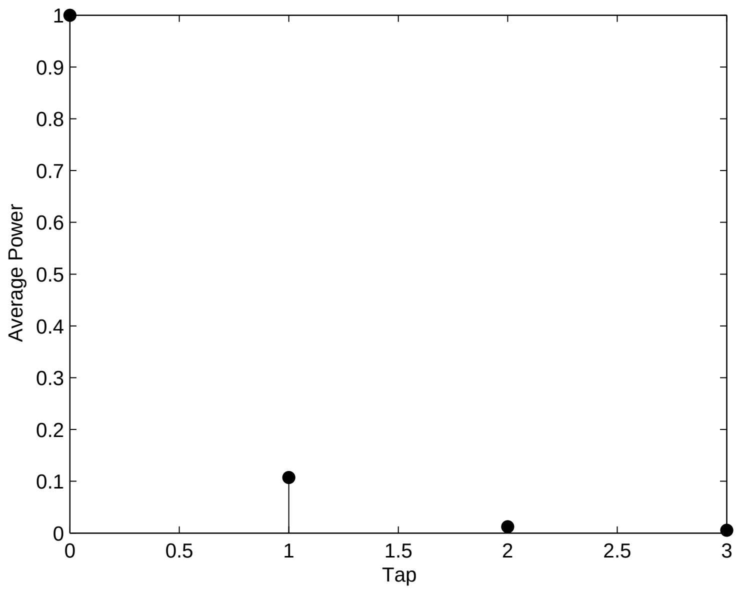

Figure 3.3: pedA channel power delay profile

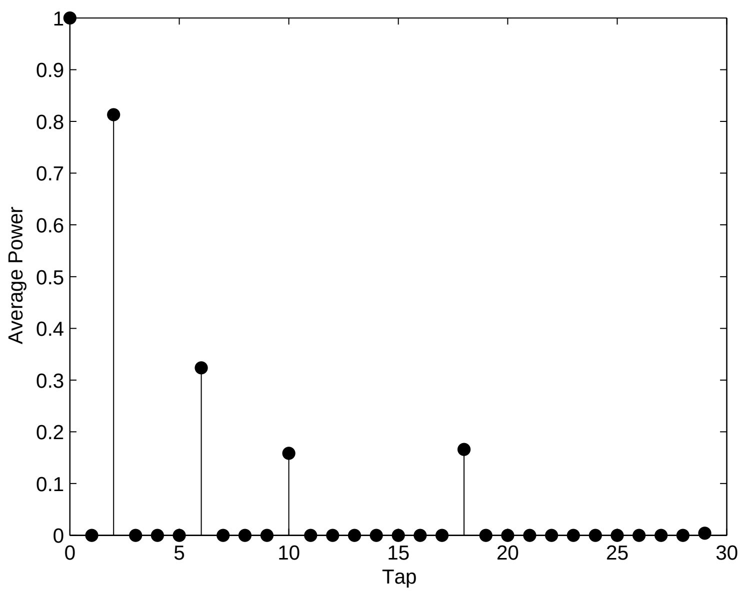

Figure 3.4: pedB channel power delay profile

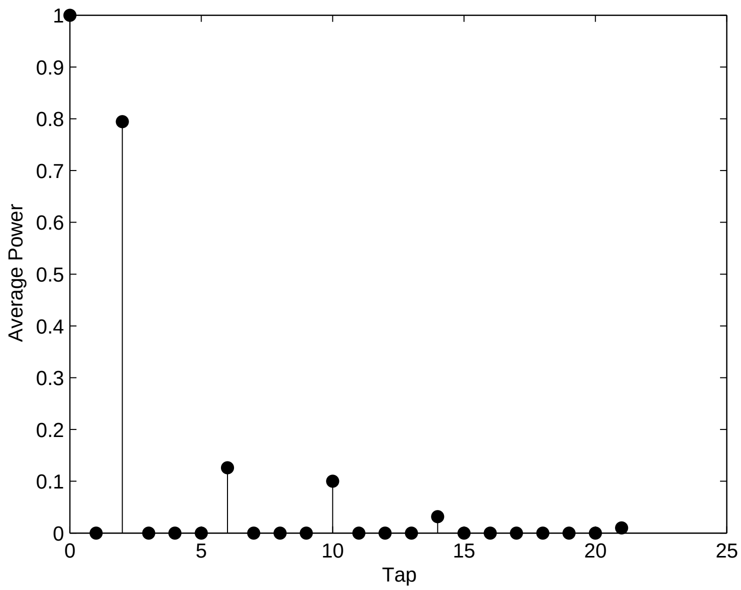

Figure 3.5: vehA channel power delay profile

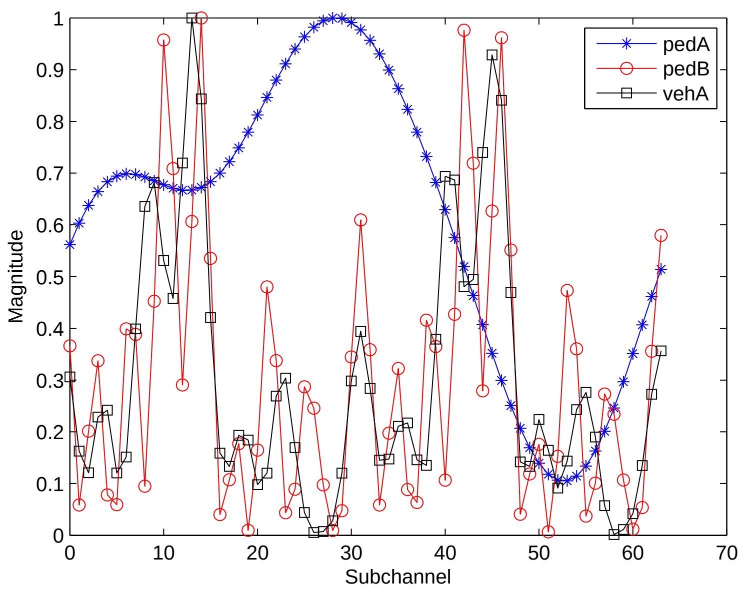

Figure 3.6: Frequency response of the channels

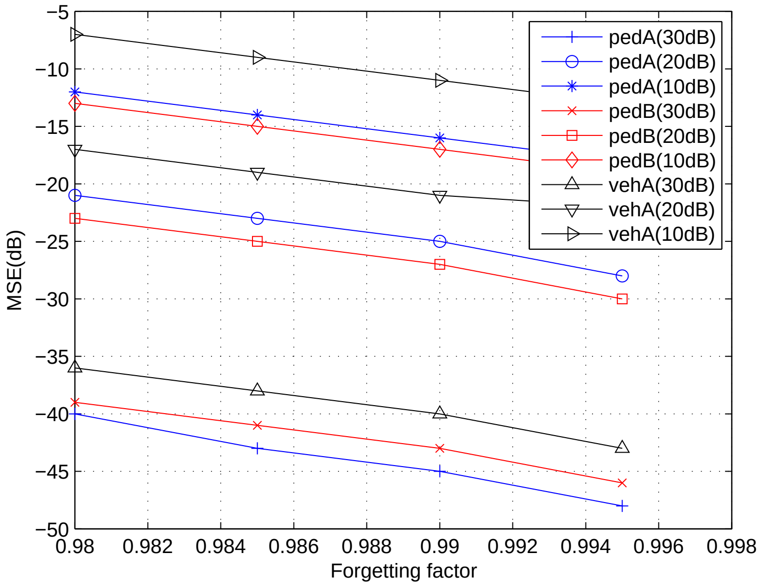

Figure 3.7: Dependence of channel estimation MSE and forgetting factor for FAM 2D-RLS

vehicular-A (vehA) channel model is used to evaluate the channel estimator performance. It is assumed that the nodes in pedA and pedB move with a velocity of 5 km/hr and 15 km/hr respectively while the nodes have a velocity of 60 km/hr in the vehA environment. Channel bandwidth is 8MHz and carrier frequency is 2.5 GHz. The power delay profiles of these channels are respectively depicted in Figure 3.3,3.4 and 3.5. Sufficient channel prefix (CP) is added such that ISI is eliminated. In the case of pedA channel CP is 3, for pedB channel CP is 29 and for vehA channel CP is 20. The forgetting factor of FAM 2D-RLS, $\lambda = 0.995$ and step size $\mu = 0.001$ for 2D-NLMS.

3.6 Results and Analysis

In Figure 3.6, the frequency response of pedA, pedB and vehA channels are shown. It is observed that pedB and vehA are highly frequency selective compared to pedA channel. In Figure 3.7, the dependence of MSE for varying values of forgetting factor is shown. It is observed that $\lambda = 0.995$ gives the best result. The computational complexity of various 2D adaptive filters are shown in Table 3.1. The computational complexity of BFAM 2D-RLS is only $O(\frac{NK^2}{M})$ while that of FAM 2D-RLS is $O(NK^2)$ and that of 2D-RLS is $O(N^2K^3)$, where K is the channel frequency response length and N is the number of OFDM samples for which the channel is correlated. In Figure 3.8,3.9 the convergence performance of FAM 2D-RLS and 2D-NLMS for estimating $\mathbf{h}^{(1)}$ and $\mathbf{g}^{(1)}$ in pedA environment is shown for varying SNR. Similarly convergence performance of the two filters are shown for pedB and vehA environment in Figure 3.10,3.11

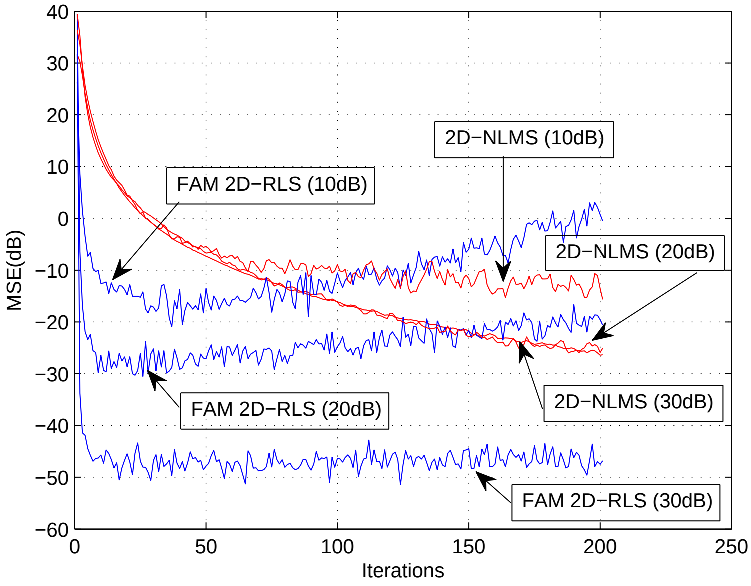

Figure 3.8: Convergence performance of FAM 2D-RLS and 2D-NLMS for estimating $\mathbf{h}^{(1)}$ in the case of a pedA channel

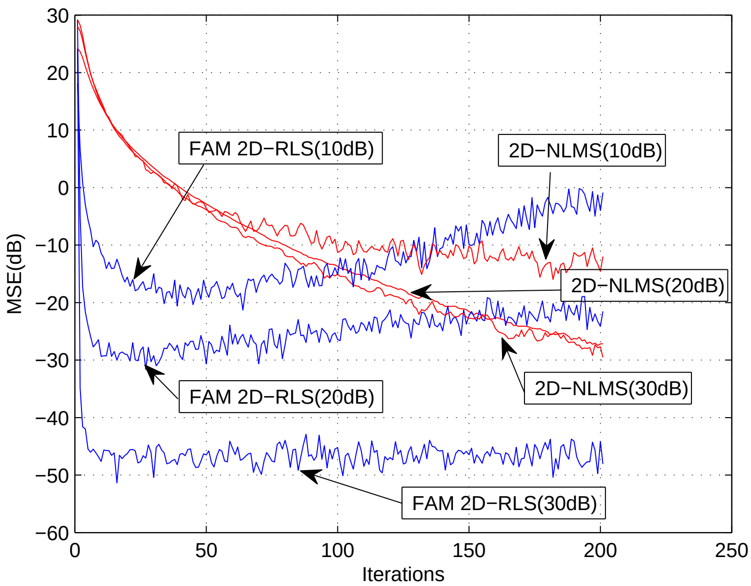

Figure 3.9: Convergence performance of FAM 2D-RLS and 2D-NLMS for estimating $\mathbf{g}^{(1)}$ in the case of a pedA channel

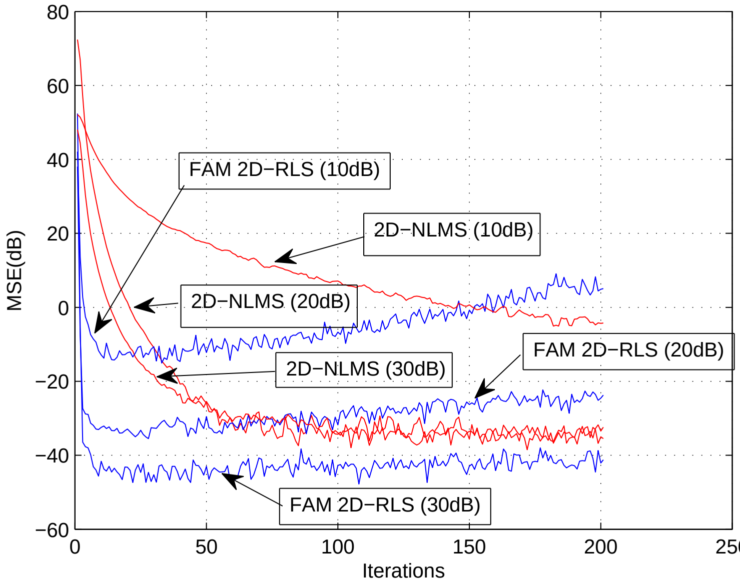

Figure 3.10: Convergence performance of FAM 2D-RLS and 2D-NLMS for estimating $\mathbf{h}^{(1)}$ in the case of a pedB channel

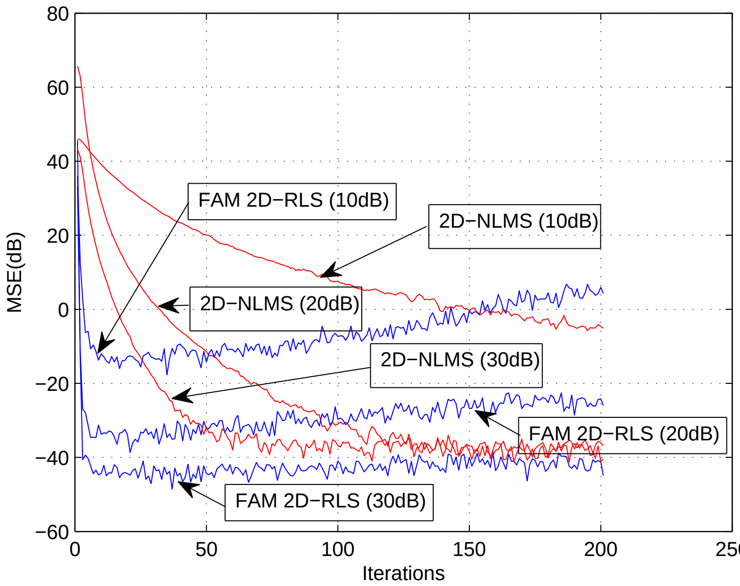

Figure 3.11: Convergence performance of FAM 2D-RLS and 2D-NLMS for estimating $\mathbf{g}^{(1)}$ in the case of a pedB channel

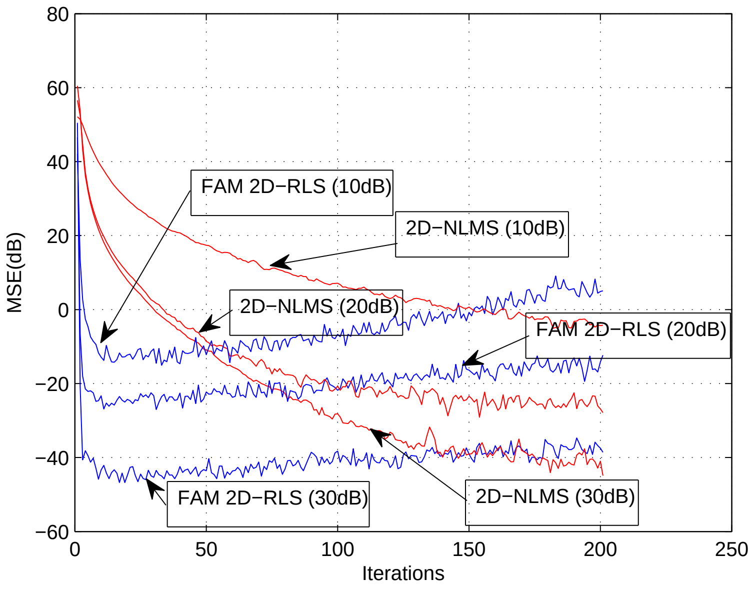

Figure 3.12: Convergence performance of FAM 2D-RLS and 2D-NLMS for estimating $\mathbf{h}^{(1)}$ in the case of a vehA channel

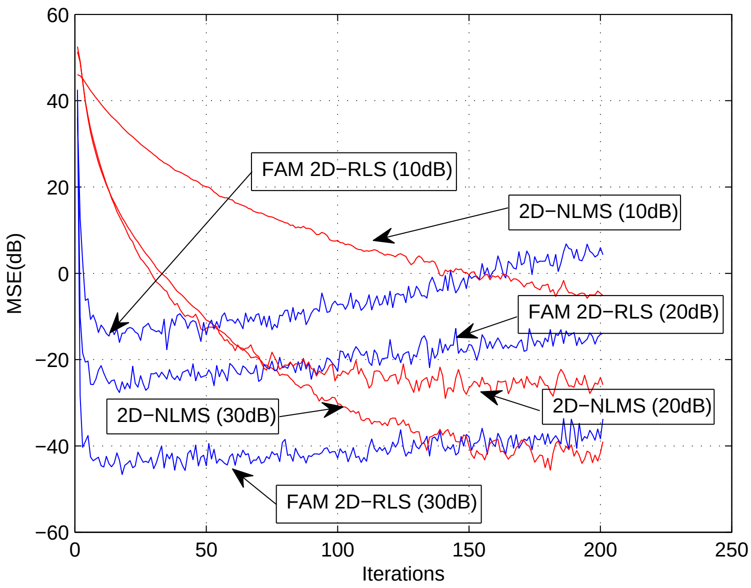

Figure 3.13: Convergence performance of FAM 2D-RLS and 2D-NLMS for estimating $\mathbf{g}^{(1)}$ in the case of a vehA channel

M	MSE(dB) at SNR=30dB	MSE(dB) at SNR=20dB	MSE(dB) at SNR=10dB
1	-48	-28	-18
2	-49	-29	-19
4	-46	-27	-17
8	-35	-19	-9
16	-25	-13	-2
32	-18	-10	does not converge

Table 3.2: Performance of BFAM 2D-RLS for varying M in case of pedA channel

M	MSE(dB) at SNR=30dB	MSE(dB) at SNR=20dB	MSE(dB) at SNR=10dB
1	-46	-30	-19
2	-46	-30	-19
4	-46	-30	-19
8	-47	-31	-19
16	-48	-31	-20
32	-49	-33	-21

Table 3.3: Performance of BFAM 2D-RLS for varying M in case of pedB channel

and Figure 3.12,3.13 respectively. It is seen that FAM 2D-RLS converges much faster than 2D-NLMS in all the cases. In Table 3.2, the MSE performance of BFAM 2D-RLS is shown for varying number of blocks M in a pedA channel environment. The MSE increases as M increases. This is because pedA is not highly frequency selective as observed in Figure 3.6. Hence when we break up the channel frequency response vector into blocks, the whole 2D correlation is not taken into consideration. In Table 3.3 and Table 3.4 the MSE performance of BFAM 2D-RLS for pedB and vehA channel is shown respectively. It is observed that MSE decreases as M increases. This is due to the fact that pedB and vehA are highly frequency selective channels and hence the channel frequency response is correlated only across a few frequency samples. Hence we note that BFAM 2D-RLS becomes effective for the case of highly frequency selective channels like pedB and vehA. We also performed the simulation for 4-QAM, 32-QAM and 64-QAM based OFDM systems. The results obtained are similar to that obtained for the 16-QAM based OFDM system. This is due to fact that we have assumed in this thesis that the symbol detection is perfect. If we account for the detection error , then the performance of the different M-ary QAM systems might be different.

In Figure 3.13 the convergence of FAM 2D-RLS is compared with 2D-NLMS for estimating $\mathbf{h}^{(1,1)}$ i.e. channel between S_1 and first antenna of relay for case of two-way relay with relay capability. SNR is 30dB. The channel is assumed to be vehA at 60 km/hr. As expected, FAM 2D-RLS outperforms 2D-NLMS. We do not perform further analysis of this estimation problem because as mentioned before, the channel estimation problem for two-way relay with relay capability is exactly same as MIMO channel estimation.

M	MSE(dB) at SNR=30dB	MSE(dB) at SNR=20dB	MSE(dB) at SNR=10dB
1	-43	-22	-13
2	-43	-23	-13
4	-43	-26	-13
8	-44	-30	-14
16	-46	-31	-16
32	-47	-33	-17

Table 3.4: Performance of BFAM 2D-RLS for varying M in case of vehA channel

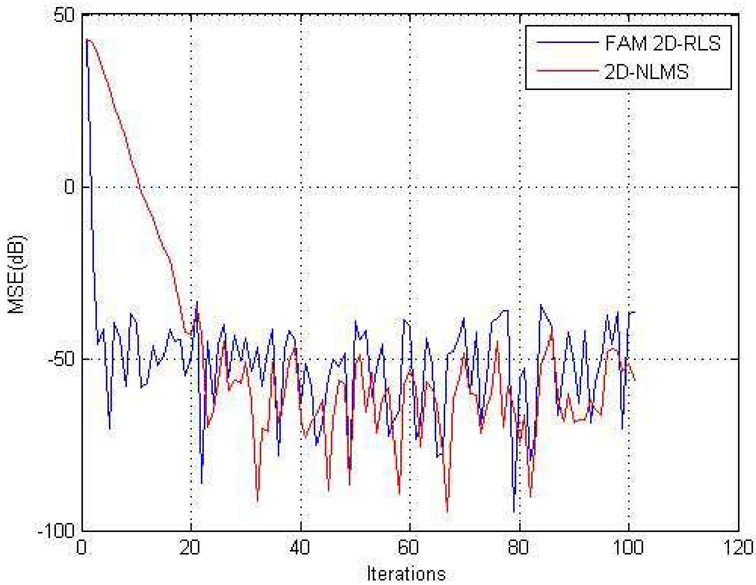

Figure 3.14: Convergence performance of FAM 2D-RLS & 2D-NLMS for estimating $\mathbf{h}^{(1,1)}$ at SNR = 30dB

3.7 Conclusion

In this chapter we introduced adaptive filter based channel estimation schemes for OFDM two-way relay systems with node capability. The adaptive filter used in this chapter is known as BFAM 2D-RLS which is a modified version of FAM-2D-RLS. It was shown that the MSE performance of BFAM 2D-RLS filter deteriorates significantly for the case of pedA channel as the number of blocks increases while in the case of pedB and VehA channels, MSE performance improves. Hence we conclude that BFAM 2D-RLS with large number of blocks are useful in the case of highly frequency selective channels while we have to use FAM 2D-RLS (single block BFAM 2D-RLS) or BFAM 2D-RLS with less blocks in the case of low frequency selective channels. In all cases it was observed that FAM 2D-RLS and BFAM 2D-RLS converged much faster than 2D-NLMS filter. We also introduced a FAM 2D-RLS filter estimation scheme for two-way relay with relay capability. It is observed that two-way relay with relay capability has the same structure as MIMO systems.

Chapter 4

Adaptive OFDM Based Two-Way Relay Systems

4.1 Introduction

In this chapter we introduce the concept of Adaptive OFDM (AOFDM) for two-way relay system. In an OFDM system, the channel is divided into subchannels and data symbols are sent over it. The data rate and BER performance of the system can be improved if the channel is known at the transmitter. Based on the gains of each subchannel, the modulation scheme and power can be varied [58],[59]. This is known as bit and power loading [60]. Most of the earlier works on loading were implemented for wired systems. AOFDM for wireless system was first proposed in [61]. AOFDM has been recently introduced for one way relay systems in [62],[63],[64]. Adaptive modulation based two-way relay system was first introduced in [65]. But all these works assumed that CSI is perfectly known. But in reality it is estimated [17]. Hence the effect of channel estimation error on the loading algorithms should be analyzed. Some of the research work for channel imperfection based AOFDM for non relay based systems are [66], [67] ,[68]

4.2 System Model

Consider a two-way relay system with transmitting nodes S_1 ,S_2 and relay R. Both the nodes and relay are single antenna devices. During Multiple Access (MA) phase both S_1 and S_2 send K subchannel OFDM data to R simultaneously. The Channel Prefix (CP) added is assumed to be sufficient to mitigate the effect of ISI. The frequency domain data received at the relay at time n after removal of CP is,

$$\mathbf{y}_n^{(R)} = \mathbf{X}_n^{(1)}\mathbf{c}_n^{(1)} + \mathbf{X}_n^{(2)}\mathbf{c}_n^{(2)} + \mathbf{n}_n^{(R)} \tag{4.1}$$

where $\mathbf{c}_n^{(1)} = \left[c^{(1)}(n,0), \cdots, c^{(1)}(n,K-1)\right]^T$ and $\mathbf{c}_n^{(2)} = \left[c^{(2)}(n,0), \cdots, c^{(2)}(n,K-1)\right]^T$ are the $K \times 1$ channel frequency response vector between $S_1 - R$ and $S_2 - R$ respectively. The diagonal matrices $\mathbf{X}_n^{(1)}$ and $\mathbf{X}_n^{(2)}$ consists of the data vector sent from S_1 and S_2 respectively. The AWGN noise vector at relay is $\mathbf{n}_n^{(R)}$. The relaying scheme is considered to be AF, hence $\mathbf{y}_n^{(R)}$ is scaled by an amplifying factor α_n as defined in the previous chapter. This amplified data is then broadcasted by the relay. The data received at S_1 and S_2 are respectively given as,

$$\mathbf{y}_n^{(1)} = \alpha_n(\mathbf{X}_n^{(1)}\mathbf{h}_n^{(1)} + \mathbf{X}_n^{(2)}\mathbf{g}_n^{(1)}) + \mathbf{w}_n^{(1)} \tag{4.2}$$

and

$$\mathbf{y}_n^{(2)} = \alpha_n(\mathbf{X}_n^{(2)}\mathbf{h}_n^{(2)} + \mathbf{X}_n^{(1)}\mathbf{g}_n^{(2)}) + \mathbf{w}_n^{(2)} \tag{4.3}$$

where $\mathbf{h}_n^{(1)} = \left[h^{(1)}(n,0), \cdots, h^{(1)}(n,K-1)\right]^T = \mathbf{c}_n^{(1)} \odot \mathbf{c}_n^{(1)}, \mathbf{g}_n^{(1)} = \mathbf{g}_n^{(2)} = \mathbf{c}_n^{(1)} \odot \mathbf{c}_n^{(2)}$ and $\mathbf{h}_n^{(2)} = \left[h^{(2)}(n,0), \cdots, h^{(2)}(n,K-1)\right]^T = \mathbf{c}_n^{(2)} \odot \mathbf{c}_n^{(2)}$. Since $\mathbf{g}_n^{(1)} = \mathbf{g}_n^{(2)}$, we ignore the superscript, and this channel is equal to $\mathbf{g}_n = [g(n,0), \cdots, g(n,K-1)]^T$. If $\mathbf{h}_n^{(1)}, \mathbf{g}_n$ and $\mathbf{h}_n^{(2)}, \mathbf{g}_n$ is known respectively at S_1 and S_2 then the self interference terms can be subtracted. After self interference cancellation the available data is,

$$\mathbf{s}_n^{(1)} = \alpha_n \mathbf{X}_n^{(2)} \mathbf{g}_n + \mathbf{w}_n^{(1)} \tag{4.4}$$

$$\mathbf{s}_n^{(2)} = \alpha_n \mathbf{X}_n^{(1)} \mathbf{g}_n + \mathbf{w}_n^{(2)} \tag{4.5}$$

The instantaneous SNR at k^{th} subchannel of S_1 and S_2 is respectively given as,

$$SNR^{(1)}(n,k) = \frac{\alpha_n^2 |g(n,k)|^2 E^{(2)}(n,k)}{\alpha_n^2 |c^{(1)}(n,k)|^2 N^{(R)} + N^{(1)}} \tag{4.6}$$

$$SNR^{(2)}(n,k) = \frac{\alpha_n^2 |g(n,k)|^2 E^{(1)}(n,k)}{\alpha_n^2 |c^{(2)}(n,k)|^2 N^{(R)} + N^{(2)}} \tag{4.7}$$

where $E^{(1)}(n,k)$ and $E^{(2)}(n,k)$ are the energy provided to k^{th} subchannel at S_1 and S_2 respectively. The noise power at S_1 is $N^{(1)}$ while that at S_2 is $N^{(2)}$. The noise power at R is $N^{(R)}$. The instantaneous sum rate of a two-way relay system is defined as,

$$R_n = \frac{1}{2}\left(\sum_{k=0}^{K-1} log_2\left(1 + SNR^{(1)}(n,k)\right) + log_2\left(1 + SNR^{(2)}(n,k)\right)\right) \tag{4.8}$$

It is observed that for a two-way relay the rate of the system consists of the sum of rates obtained at S_1 and S_2. The pre-log factor of $\frac{1}{2}$ is due to the fact that two time slots are required in order for the overall communication between S_1 and S_2 to be completed i.e, data is sent from S_1

and S_2 simultaneously to the relay in the first time slot while in the next time slot the relay broadcasts an amplified version of this data to the nodes. The pre-log factor should be taken into consideration while designing the adaptive OFDM algorithm. Since the equation of data rate in (4.8) is different from the equation of data rate of a non relay system or one-way relay, some modifications have to be made in the derivation of existing loading algorithms.

4.3 Obtaining Individual CSI from Overall CSI

Implementing adaptive OFDM requires the knowledge of $SNR^{(2)}(n,k)$ and $SNR^{(1)}(n,k)$ at S_1, S_2 respectively. It is observed in (4.6) and (4.7) that $SNR^{(1)}$ and $SNR^{(2)}$ can be only calculated if the individual channels $\mathbf{c}^{(2)}$ and $\mathbf{c}^{(1)}$ are known at S_1 and S_2. It was shown in the previous chapter that DDCE adaptive filter based channel estimator could be designed to estimate $\mathbf{h}_n^{(1)}, \mathbf{g}_n$ at S_1 and $\mathbf{h}^{(2)}, \mathbf{g}_n$ at S_2. We know that $\mathbf{h}^{(1)} = \mathbf{c}^{(1)} \odot \mathbf{c}^{(1)}$ and $\mathbf{g}^{(1)} = \mathbf{c}^{(1)} \odot \mathbf{c}^{(2)}$. Let's consider the k^{th} frequency coefficient $h^{(1)}(n,k) = (c^{(1)}(n,k))^2$ at S_1. The individual channel can be obtained with a sign ambiguity by taking the square root.

$$\sqrt{h^{(1)}(n,k)} = \pm c^{(1)}(n,k) \tag{4.9}$$

We may choose $+$ or $-$ sign. Since $g(n,k)$ is available, we can obtain $c(n,k)$ as,

$$\frac{g(n,k)}{\pm c^{(1)}(n,k)} = \pm c^{(2)}(n,k) \tag{4.10}$$

Similarly $c^{(1)}(n,k)$ can be calculated at S_2. It must be noted that the sign ambiguity only appears in pairs, i.e if $-c^{(1)}(n,k)$ is selected when in reality the channel is $+c^{(1)}(n,k)$, then we obtain $-c^{(2)}(n,k)$. This is known as simultaneous sign ambiguity [9], [69]. The sign ambiguity will not be a problem because the SNR calculation requires only the knowledge of magnitude of channel coefficient.

4.4 Adaptive OFDM for Two-Way Relay Systems

The channel capacity of k^{th} channel of S_1 is defined as,

$$R^{(1)}(n,k) = \frac{1}{2} log_2 \left(1 + SNR^{(2)}(n,k)\right) \tag{4.11}$$

This is the maximum rate that can be transmitted assuming that the probability of error tends to zero. In practice the probability of error is greater than zero. Thus actual number of bits that can be transmitted per QAM symbol on the k^{th} subchannel is,

$$b^{(1)}(n,k) = \frac{1}{2}log_2\left(1 + \frac{SNR^{(2)}(n,k)}{\Gamma}\right) \tag{4.12}$$

where Γ is the gap function [19]. The equal probability of error at S_2 for each subchannel is assumed to be $P_e^{(2)}$. The gap function can be calculated as [19],

$$\Gamma = \frac{1}{3}\left[Q^{-1}\left(\frac{b^{(1)}(n,k)P_e^{(2)}}{4(1-2^{-b^{(1)}(n,k)/2})}\right)\right]^2 \tag{4.13}$$

Let $SNR^{(2)}(n,k) = \frac{E^{(1)}(n,k)}{CNR^{(2)}(n,k)}$ where,

$$CNR^{(2)}(n,k) = \frac{\alpha^2|g^{(1)}(n,k)|^2}{\alpha^2|c^{(2)}(n,k)|^2N^{(R)} + N^{(2)}} \tag{4.14}$$

then (4.12) can be rewritten as,

$$b^{(1)}(n,k) = \frac{1}{2}log_2\left(1 + \frac{E^{(1)}(n,k)CNR^{(2)}(n,k)}{\Gamma}\right) \tag{4.15}$$

Hence the instantaneous energy applied to k^{th} subchannel at S_1 is,

$$E^{(1)}(n,k) = \frac{\Gamma}{CNR^{(2)}(n,k)}(2^{2b^{(1)}(n,k)} - 1) \tag{4.16}$$

The amount of extra energy needed to add a single bit to the k^{th} subchannel is ,

$$\begin{aligned} E^{(1)}_{b(n,k)+1} - E^{(1)}_{b(n,k)} &= \tfrac{\Gamma}{CNR^{(2)}(n,k)}(2^{2(b(n,k)+1)} - 1) - \tfrac{\Gamma}{CNR^{(2)}(n,k)}(2^{2b(n,k)} - 1) \\ &= \tfrac{3\Gamma}{CNR^{(2)}(n,k)}2^{2b(n,k)} \end{aligned} \tag{4.17}$$

where $E^{(1)}_{b(n,k)}$ is the amount of energy required to transmit $b^{(1)}(n,k)$ bits from S_1 at a given probability of error. The LHS of above equation is known as incremental energy. The loading algorithm assigns bits to the subchannel having the least incremental energy.

4.4.1 Levin-Campello Loading Algorithm

In [20] Campello proposed the necessary and sufficient conditions for a bit distribution to be a solution of Margin Adaptive (MA) and Rate Adaptive (RA) loading algorithms. Assume that we have distributed the bits according to a vector $\mathbf{b}_n^{(1)} = \left[b^{(1)}(n,0)\cdots b^{(1)}(n,K-1)\right]^T$. This allocation is considered to be energy efficient if,

$$\underset{i}{max}\ \Delta E^{(1)}_{b(n,i)} \leq \underset{j}{min}\ \Delta E^{(1)}_{b(n,j)+1} \quad i,j = 0,\cdots,K-1. \tag{4.18}$$

where $\Delta E^{(1)}_{b(n,j)+1}$ is the incremental energy as defined in (4.17). If any random bit allocation is provided then it can be converted to an efficient allocation by performing the efficientizing algorithm [20] as given below,

Step-1: Find $\underset{i}{max}\ \Delta E^{(1)}_{b(n,i)}$ and $\underset{j}{min}\ \Delta E^{(1)}_{b(n,j)+1}$
Step-2: Check if $\Delta E^{(1)}_{b(n,j)+1} < \Delta E^{(1)}_{b(n,i)}$, where $j, i = 0 \cdots K-1$
Step-3: If step-2 is satisfied, then one bit from subcarrier i should be transferred to j.
Step-4: Go to step-1 and then to step-2 until the condition in step-2 is not satisfied.

RA Solution

In RA solution the following optimization problem is solved,

$$\begin{aligned} &\underset{\mathbf{b}^{(1)}_n \in \mathbf{Z}^K}{max} \sum_{k=0}^{K-1} b^{(1)}(n,k) \\ subject\ to &\sum_{k=0}^{K-1} E^{(1)}(n,k) \leq E^{(1)} \end{aligned} \tag{4.19}$$

where $E^{(1)}$ is the total energy available at S_1. In order to satisfy this constraint we have to modify the efficientizing algorithm. This is known as E-tightening . A bit distribution is E-tight if,

$$\sum_{K=0}^{k-1} E^{(1)}_{b(n,k)} \leq E^{(1)} \leq \sum_{K=0}^{k-1} E^{(1)}_{b(n,k)} + \underset{0 \leq k \leq K-1}{min} \Delta E^{(1)}_{b(n,k)+1} \tag{4.20}$$

This means we can add an extra bit only if more energy than the maximum energy $E^{(1)}$ is available. The algorithm for E-tightness is given as,
Step-1: The total energy assigned to the subchannel by S_1 till the present iteration is $E^{(1)}_a = \sum_{k=1}^{K-1} E^{(1)}_{b(n,k)}$
Step-2: If $E^{(1)} - E^{(1)}_a < 0$, i.e. more energy is allocated than the maximum available, then remove a bit from subchannel having maximal incremental energy. Again check if condition is satisfied.
Step-3: If $E^{(1)} - E^{(1)}_a \geq \underset{k}{min}\ \Delta E_{b(n,k)+1}$, i.e there is still energy available to increment the number of bits. Add a bit to the subchannel requiring the minimum incremental bit energy. Perform this step until energy is allocated optimally.

MA solution

In MA solution the following optimization problem is satisfied,

$$\begin{aligned} \min_{\mathbf{b}_n^{(1)} \in \mathbf{Z}^K} & \sum_{k=0}^{K-1} E^{(1)}(n,k) \\ subject\ to & \sum_{k=0}^{K-1} b^{(1)}(n,k) = B^{(1)} \end{aligned} \tag{4.21}$$

where $B^{(1)}$ is the total bits that is to be transmitted in a single OFDM symbol from S_1. Some modifications are required to the efficientizing algorithm. This is known as B-tightening algorithm as given below,

Step-1: Let the total assigned bit be $B_a = \sum_{k=0}^{K-1} b^{(1)}(n,k)$

Step-2: If $B_a^{(1)} > B^{(1)}$, i.e. more bits are assigned than the target. In this case remove a bit from the subcarrier having maximal incremental energy.

Step-3: If $B_a^{(1)} < B^{(1)}$, then add a bit to the subcarrier having the minimal incremental energy.

It can be observed for RA and MA solution that, they both solve a similar optimization problem. This duality is explained in [70].

4.5 Channel Estimation Error and AOFDM

In the previous sections it was assumed that the perfect CSI information was available at S_1 and S_2. In reality only as estimate of CSI, which can be obtained by the method proposed in the previous chapter is available. Hence the SNR values calculated at the nodes will be different compared to (4.6), (4.7). The estimate of the channel frequency response vectors are $\hat{\mathbf{h}}_n^{(1)}, \hat{\mathbf{h}}_n^{(2)}, \hat{\mathbf{g}}_n$. They are defined as,

$$\hat{\mathbf{h}}_n^{(1)} = \mathbf{h}_n^{(1)} + \xi_{\mathbf{h}_n^{(1)}} \tag{4.22}$$

$$\hat{\mathbf{h}}_n^{(2)} = \mathbf{h}_n^{(2)} + \xi_{\mathbf{h}_n^{(2)}} \tag{4.23}$$

$$\hat{\mathbf{g}}_n = \mathbf{g}_n + \xi_{\mathbf{g}_n} \tag{4.24}$$

where $\xi_{\mathbf{h}_n^{(1)}}, \xi_{\mathbf{h}_n^{(2)}}$, and $\xi_{\mathbf{g}_n}$ are the estimation error vectors. It can be assumed to be an AWGN vector [66] with variance equal to the MSE of the channel estimation error. The MSE of channel estimation are defined as,

$$MSE_{\mathbf{h}_n^{(1)}} = E\left[\frac{\|\xi_{\mathbf{h}_n^{(1)}}\|^2}{K}\right] \quad (4.25)$$

$$MSE_{\mathbf{h}_n^{(2)}} = E\left[\frac{\|\xi_{\mathbf{h}_n^{(2)}}\|^2}{K}\right] \quad (4.26)$$

$$MSE_{\mathbf{g}_n} = E\left[\frac{\|\xi_{\mathbf{g}_n}\|^2}{K}\right] \quad (4.27)$$

In this chapter without loss of generality, we assume that $MSE_{\mathbf{h}_n^{(1)}} = MSE_{\mathbf{h}_n^{(2)}} = MSE_{\mathbf{g}_n} = MSE$. Due to the channel estimation error, the self interference cancellation defined in (4.4),(4.5) should be modified as,

$$\mathbf{s}_n^{(1)} = \alpha_n \mathbf{X}_n^{(2)} \mathbf{g}_n + \mathbf{w}_n^{(1)} - \alpha_n \mathbf{X}_n^{(1)} \xi_{\mathbf{h}_n^{(1)}} \quad (4.28)$$

$$\mathbf{s}_n^{(2)} = \alpha_n \mathbf{X}_n^{(1)} \mathbf{g}_n + \mathbf{w}_n^{(2)} - \alpha_n \mathbf{X}_n^{(2)} \xi_{\mathbf{h}_n^{(2)}} \quad (4.29)$$

Equation (4.6),(4.7) is modified as,

$$SNR^{(1)}(n,k) = \frac{\alpha_n^2 |g(n,k)|^2 E^{(2)}(n,k)}{\alpha_n^2 |c^{(1)}(n,k)|^2 N^{(R)} + N^{(1)} - \alpha^2 E^{(1)}(n,k) MSE} \quad (4.30)$$

$$SNR^{(2)}(n,k) = \frac{\alpha_n^2 |g(n,k)|^2 E^{(1)}(n,k)}{\alpha_n^2 |c^{(2)}(n,k)|^2 N^{(R)} + N^{(2)} - \alpha^2 E^{(2)}(n,k) MSE} \quad (4.31)$$

In a practical scenario this modified SNR is used for implementing AOFDM. In the next section computer simulations are performed to analyze the performance of AOFDM on two-way relay systems with channel estimation error.

4.6 Simulation setup

The channels used are pedA, pedB and vehA as discussed in previous chapter. The number of subcarriers for OFDM is 64. The total number of bits B used in MA is 256 and the total available energy at S_1, S_2 is $E^{(1)} = E^{(2)} = 30dB$. The target probability of error is 10^{-5}.

4.7 Results and Analysis

In Figure 4.1, the normalized individual pedA channels $\mathbf{c}^{(1)}$ and $\mathbf{c}^{(2)}$ is shown. The product of these two channels form the combined channel $\mathbf{g}_n$ as shown in Figure 4.2. The gain of $\mathbf{g}_n$ is used in calculating SNR. It is observed that the gain of the channel $\mathbf{g_n}$ is high only if both $\mathbf{c}^{(1)}$ and $\mathbf{c}^{(2)}$ has high gain as can be observed from Figure 4.1,4.2. In Figure 4.3,4.4 bit

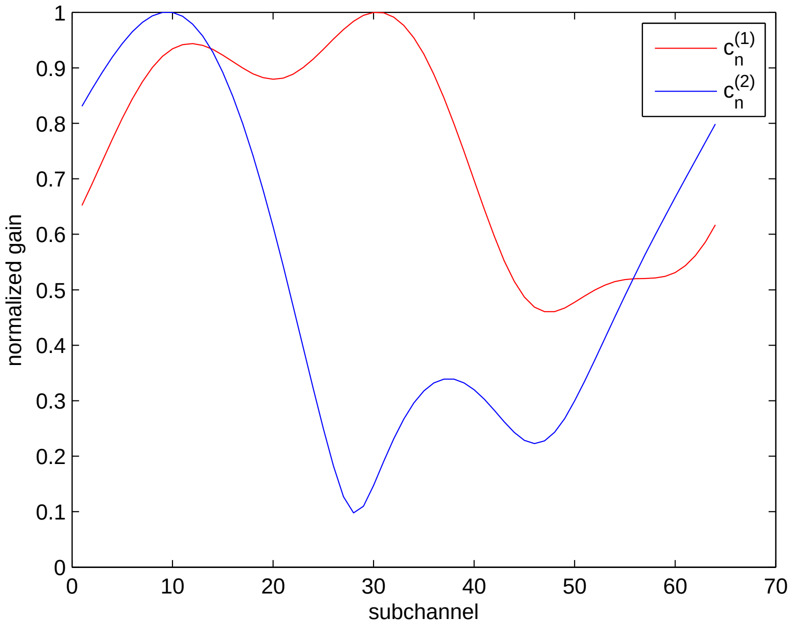

Figure 4.1: Individual pedA channels $\mathbf{c}_n^{(1)}$, $\mathbf{c}_n^{(2)}$

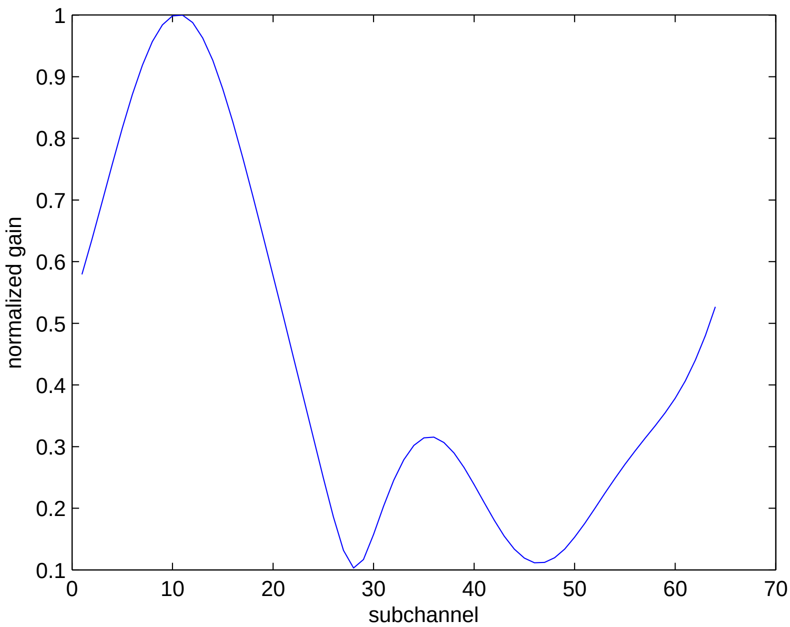

Figure 4.2: Overall pedA channel $\mathbf{g}_n$

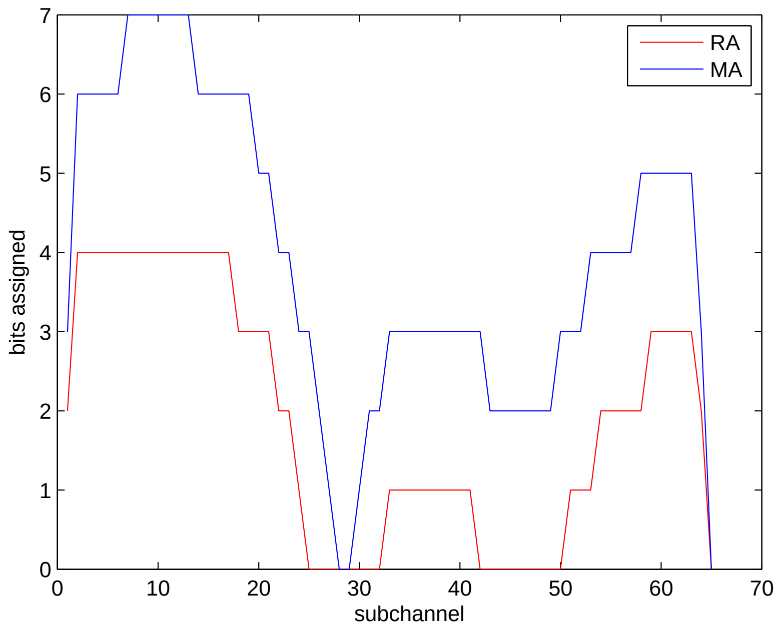

Figure 4.3: Bit allocation for RA and MA assuming perfect CSI for pedA channel

allocation and energy allocation for perfect CSI is plotted for pedA channel. Comparing with Figure 4.2, it can be observed that bit and energy is allocated based on the gain of the combined channel. When the combined subchannel gain is very low, no bit or energy is allocated to this subchannel. Hence the concept of capacity versus outage is insignificant in the case of AODFM [1]. Similarly bit and power allocation are depicted for pedB channel in Figure 4.5,4.6. The performance of the loading algorithm for vehA channel environment is shown in Figure 4.7,4.8. In Table 4.1, the total bits allocated in RA and total energy allocated in MA are shown for varying MSE of channel estimation of combined channel in pedA environment. It is seen that for $MSE \leq -15dB$, the performance of AOFDM is similar to that of a perfect CSI case. Similar observations are made for the case of pedB and vehB channels in Table 4.2,4.3. In the previous chapter it was observed that the MSE of channel estimation using FAM 2D-RLS was lesser than -15dB in all cases of SNR except for vehA channel where it was -13dB for SNR = 10dB. But when the number of blocks were increased, the MSE of BFAM 2D-RLS decreased to -17dB. In the case of pedA channel, increasing the blocks decreased the MSE to -10dB for SNR=20dB while the filter does not converge in the case of SNR = 10dB. Hence we can state that the Levin-campello based loading for two-way relay works well when we use a BFAM 2D-RLS with large number of blocks in the case of pedB and vehA channel, while the block number should be less in the case of pedA channel. BFAM 2D-RLS with $M = 1$ based was seen to provide the best estimate for implementing AOFDM in case of pedA channel while BFAM 2D-RLS with M=32 gives the best estimate for implementing AOFDM in case of pedB,

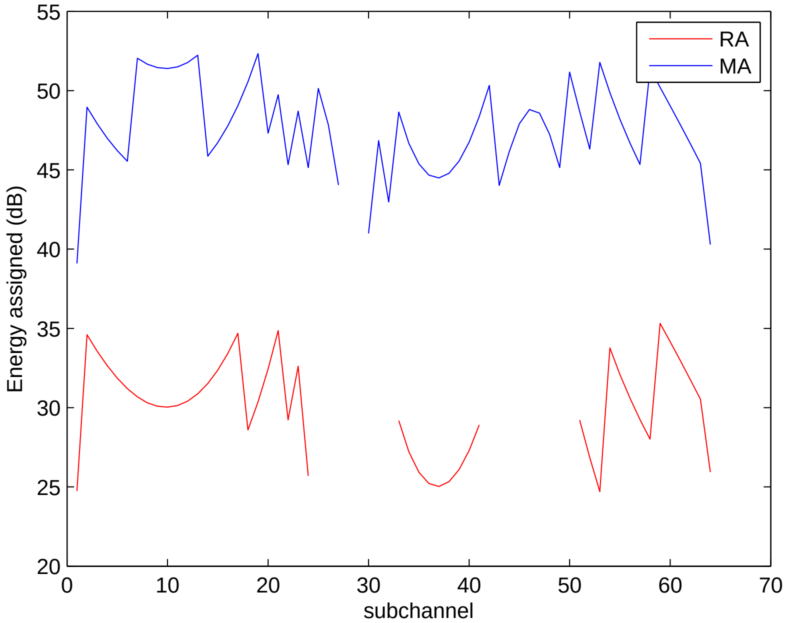

Figure 4.4: Energy allocation for RA and MA assuming perfect CSI for pedA channel

MSE (dB)	Bits assigned (RA)	Energy assigned in dB (MA)
perfect CSI	122	38.7701
-30	122	38.7703
-20	121	38.7712
-15	120	38.7741
-10	103	39.1032
0	81	39.9241

Table 4.1: Performance of Levin-Campello RA/MA loading algorithm for varying values of MSE of pedA channel estimation for two-way relay

vehA channel.

4.8 Conclusion

In this chapter we implemented an AOFDM based two-way relay system. A simple method is proposed to obtain the individual channels from the combined channel of a two-way relay. This CSI is then used to calculate the SNR. The AOFDM is based on Levin-Campello algorithm. But our framework can be used to implement any other AOFDM schemes for the case of two-way relays. The effect of CSI estimation error on performance of AOFDM is analyzed. It is observed that $MSE \leq -15dB$ gives similar performance as that of perfect CSI case. Hence if we have a channel estimator with $MSE \leq -15dB$, then an AOFDM based scheme can be used for two-way relay without loss of performance. In the case of a time varying channel, the gain varies

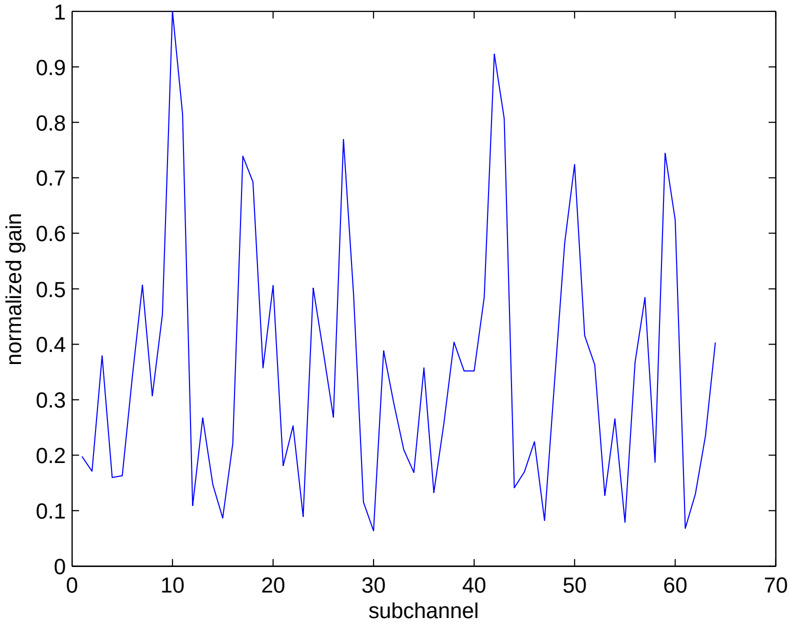

Figure 4.5: Overall pedB channel g_n

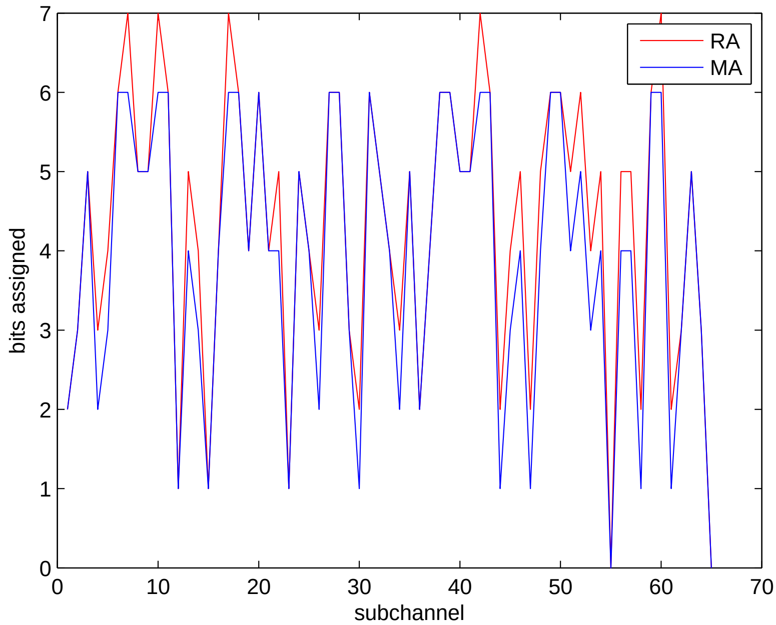

Figure 4.6: Bit allocation for RA and MA assuming perfect CSI for pedB channel

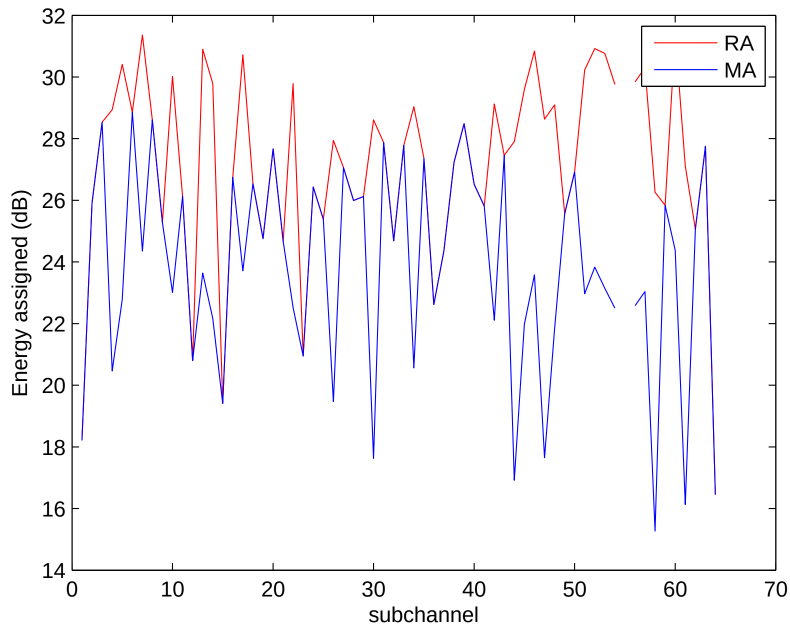

Figure 4.7: Energy allocation for RA and MA assuming perfect CSI for pedB channel

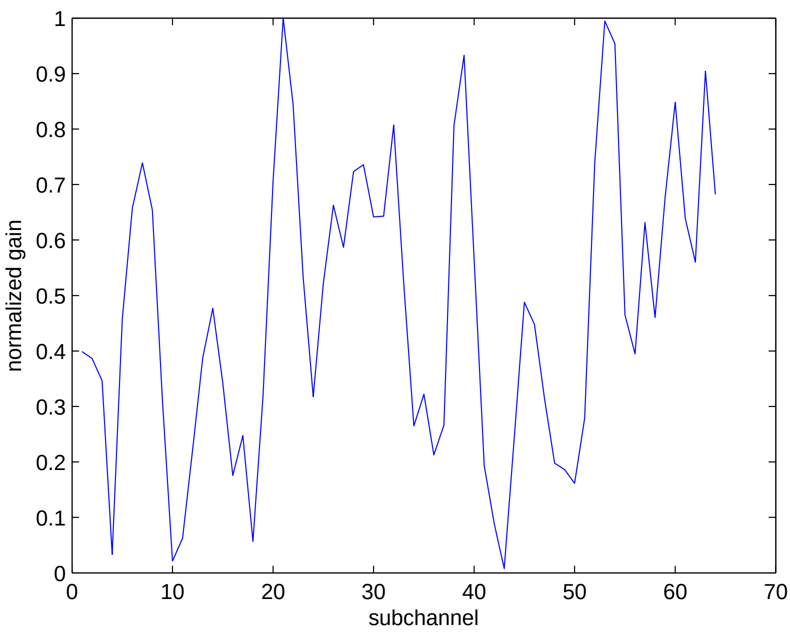

Figure 4.8: Overall vehA channel g_n

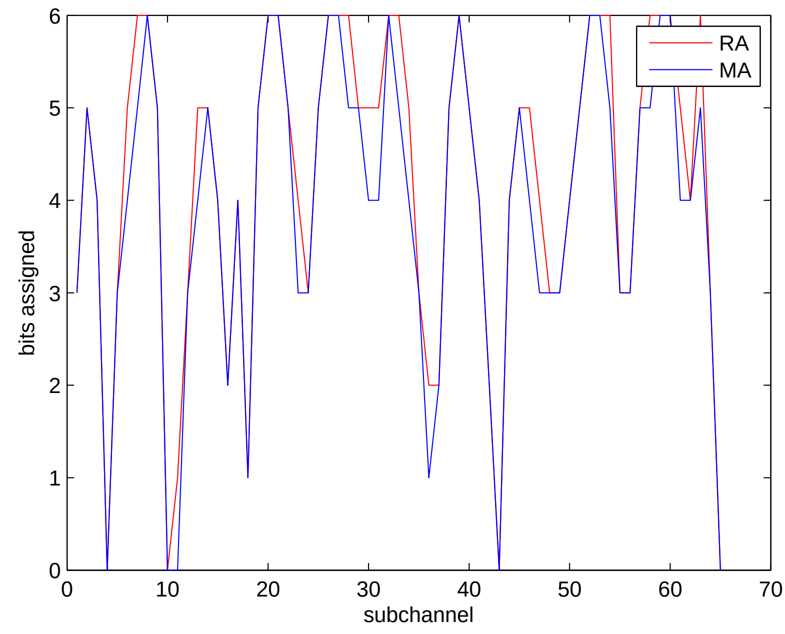

Figure 4.9: Bit allocation for RA and MA assuming perfect CSI for vehA channel

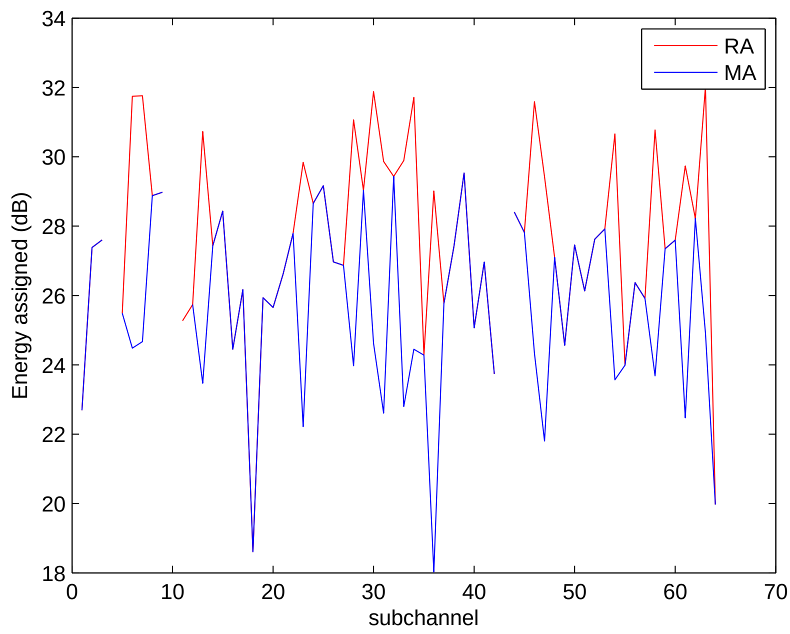

Figure 4.10: Energy allocation for RA and MA assuming perfect CSI for vehA channel

MSE (dB)	Bits assigned (RA)	Energy assigned in dB (MA)
perfect CSI	282	28.5621
-30	282	28.5621
-20	280	28.5563
-15	279	28.5416
-10	261	29.1033
0	243	29.7243

Table 4.2: Performance of Levin-Campello RA/MA loading algorithm for varying values of MSE of pedB channel estimation for two-way relay

MSE (dB)	Bits assigned (RA)	Energy assigned in dB (MA)
perfect CSI	273	27.0564
-30	273	27.0565
-20	271	27.0572
-15	270	27.0612
-10	263	28.1014
0	234	29.0065

Table 4.3: Performance of Levin-Campello RA/MA loading algorithm for varying values of MSE of vehA channel estimation for two-way relay

w.r.t time. This can be tracked with the help of an adaptive filter. Hence our proposed method for adaptive channel estimation in the previous chapter can be used for estimating and tracking the channel, followed by implementation of an AOFDM scheme. In the case of pedA channel the number of blocks for BFAM 2D-RLS should be less so that the condition of $MSE \leq -15dB$ is satisfied for low SNR. In the case of pedB and vehA channels the number of blocks of BFAM 2D-RLS should be high so that the condition of $MSE \leq -15dB$ is satisfied for low SNR.

Chapter 5

Conclusion

In this thesis a low complexity adaptive DDCE based channel estimation is proposed for two-way relay systems. The low complexity channel estimator called FAM 2D-RLS filter has a computational complexity comparable to that of 2D-NLMS scheme while having similar convergence performance of the classic 2D-RLS algorithm. The steady state analysis of an adaptive filter with weight matrix is derived based on the fact that any adaptive filter can be considered as an iterative equation solver with RLS being a special case. Hence our method can be used for analyzing any adaptive filter with weight matrix. The channel estimation can either be performed at the relay or at the node. FAM 2D-RLS channel estimation for both these schemes are proposed. The convergence performance is analyzed by varying the SNR value. The computational complexity of FAM 2D-RLS is further reduced. This adaptive filtering scheme is called BFAM 2D-RLS. The BFAM 2D-RLS filter consists of M parallel FAM 2D-RLS filters that estimates the channel frequency response vector at each iteration. It is seen that BFAM 2D-RLS has computational complexity lesser that FAM 2D-RLS by a factor of $\frac{1}{M}$. The MSE performance of BFAM 2D-RLS increases as M is increased for pedA channel while it decreases for pedB and vehA channels. The estimated channel is used for implementing a bit/power loading algorithm for two-way relay. A simple technique for separating the combined channel is proposed so that the SNR can be calculated. The loading algorithm used is the Levin-Campello algorithm. But our framework could be used for implementing other loading algorithms for two-way relay systems. It is observed that the loading algorithm with channel estimation error of $MSE \leq -15dB$ has similar performance as perfect CSI case.

The future work comprises of implementing the FAM 2D-RLS based channel estimation technique in a cooperative communication scheme. Hence it finds application in a decentralized cognitive relay network.

Appendix A

OFDM

In a wireless communication scenario a symbol transmitted will be reflected by scatterers like trees or buildings present in between the transmitter and receiver. Due to the presence of multiple scatterers, we obtain multiple copies of the symbol. This is known as the multipath effect. Let us consider a simple two path channel with channel coefficient h_0 and h_1. If $x(0), x(1)$ are the symbols transmitted, then the received symbols would be,

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \end{bmatrix} = \begin{bmatrix} h_0 & 0 \\ h_1 & h_0 \\ 0 & h_0 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \end{bmatrix} + \begin{bmatrix} n(0) \\ n(1) \\ n(2) \end{bmatrix} \tag{A.1}$$

where the coefficients n(.) comprises of Additive White Gaussian Noise (AWGN). The above equation can be written as $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$. The channel estimation problem comprises of finding the coefficients of $\mathbf{H}$ by observing $\mathbf{y}$. After $\mathbf{H}$ is estimated the effect of the channel could be removed by multiplying both sides of the equation by $\mathbf{H}^{-1}$. The complexity involved in inverting this matrix could be drastically reduced if $\mathbf{H}$ can be converted into a diagonal matrix. OFDM technique can be used to diagonalize the channel matrix. Initially the data symbols to be transmitted are converted into groups of K. This $K \times 1$ vector is known as transmission vector and is denoted as $\mathbf{x}$. The K point Inverse Discrete Fourier Transform (IDFT) of this vector is performed. Let it be denoted as $\mathbf{s} = \mathbf{W}^H\mathbf{x}$, where $\mathbf{W}$ is the K point Discrete Fourier Transform (DFT) matrix. Assuming that the channel length L is known, the last $L - 1$ frequency samples of $\mathbf{s}$ are appended to the beginning of $\mathbf{s}$. This redundant data is known as Cyclic Prefix (CP). So the new transmission vector becomes $\mathbf{s}_{cp}$ and is of dimension $K + L - 1 \times 1$. At the receiver the CP data is removed . Hence the received data is $\mathbf{r} = \mathbf{H}\mathbf{W}^H\mathbf{x} + \mathbf{n}$. It is seen that $\mathbf{H}$ is a circulant matrix. Hence we can use a result from matrix theory which states that any circulant matrix is diagonalized by the DFT matrix. Hence the channel matrix is diagonalized by $\mathbf{W}\mathbf{H}\mathbf{W}^H = \Lambda$, where Λ is the diagonal matrix that contains the DFT coefficients of the channel coefficients. In order to bring in this notion of diagonalizing the channel matrix, we perform a K point DFT of the received data vector $\mathbf{r}$ as shown,

$$\mathbf{Wr} = \mathbf{WHW}^H\mathbf{x} + \mathbf{Wn} = \mathbf{\Lambda x} + \mathbf{Wn} \tag{A.2}$$

Hence OFDM channel estimation in the frequency domain is the problem of estimating the diagonal elements in $\mathbf{\Lambda}$ by observing $\mathbf{Wr}$. This could be estimated by making use of an adaptive filter.

Appendix B

Adaptive Filter

Assume the scenario of estimating a scalar zero mean random variable d by observing a zero mean random vector $\mathbf{u}$. The observation is assumed to be corrupted by noise. The dimension of $\mathbf{u}$ is $1 \times M$. In the standard literature on adaptive filter [24], d is known as desired data and $\mathbf{u}$ is the observation vector. If $\mathbf{w}$ is the weight matrix of dimension $M \times 1$, then the estimation problem is to find a linear estimate of d such that the following cost function is minimized,

$$J(\mathbf{w}) = E\left[|d - \mathbf{u}\mathbf{w}|^2\right] \tag{B.1}$$

where E is expectation operator. This cost function is know as the mean square error. In order to minimize this function , the derivate with respect to $\mathbf{w}$ is taken and is then equated to zero. The solution thus obtained is $\mathbf{w}^o$ and is defined as,

$$\mathbf{w}^o = \mathbf{R}_u^{-1}\mathbf{R}_{du} \tag{B.2}$$

where $\mathbf{R}_u = E\left[\mathbf{u}^H\mathbf{u}\right]$ is the covariance matrix of $\mathbf{u}$ and $\mathbf{R}_{du} = E\left[d\mathbf{u}^H\right]$ is the cross covariance vector of d and $\mathbf{u}$. Unlike the case of mean square error, the closed form solution i.e $\mathbf{w}^o$ might not exist for general cost function [24]. It is also observed in (B.2) that, the inverse of the $M \times M$ matrix $\mathbf{R}_u$ is to be performed. It has a computational cost of $O(M^3)$. In order to alleviate these problems, we could use an iterative method to solve the linear equation. One of the common iterative method is known as the steepest descent method. The basic idea of this method is that at every iteration i a solution $\mathbf{w}_i$ is obtained from the solution at time $i-1$. The weight update equation is,

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu\mathbf{p} \tag{B.3}$$

where μ is the step size and $\mathbf{p}$ is the negative of the gradient of cost function. The reason for choosing the negative gradient is that it points towards the decreasing direction of the curve of the cost function. Hence for the case of a cost function with a global minimum, if a proper step size is chosen the solution converges to the optimal value . The mean square error cost function could be iteratively minimized by the following update equation [24],

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu \left[\mathbf{R}_{du} - \mathbf{R}_u \mathbf{w}_{i-1} \right] \quad i \geq 0 \tag{B.4}$$

In order to perform the steepest descent algorithm we require the knowledge of exact statistics i.e. $\mathbf{R}_{du}$ and $\mathbf{R}_u$. But it is rarely known in practice. Hence approximations of the covariance matrices are made. Based on the approximations made, two of the commonly used adaptive filter algorithms are Normalized Least Mean Square (NLMS) and Recursive Least Square Algorithm (RLS).

B.1 NLMS Algorithm

NLMS algorithm is a modified version of the Least Mean Square(LMS) algorithm. The approximations of the covariance matrices at each iteration are $\mathbf{R}_{du,i} = d(i)\mathbf{u}_i^H$ and $\mathbf{R}_{u,i} = \mathbf{u}_i^H \mathbf{u}_i$. Substituting this in (B.4), the LMS weight update equation is,

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu \mathbf{u}_i^H \left[d(i) - \mathbf{u}_i \mathbf{w}_{i-1} \right] \tag{B.5}$$

The above equation is modified to obtain the NLMS update equation as,

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \frac{\mu}{\epsilon + \|\mathbf{u}_i\|^2} \mathbf{u}_i^H \left[d(i) - \mathbf{u}_i \mathbf{w}_{i-1} \right] \tag{B.6}$$

The reason for this modification is that in LMS the difference between $\mathbf{w}_i$ and $\mathbf{w}_{i-1}$ depend on the norm of $\mathbf{u}_i$. So when $\mathbf{u}_i$ has a large norm, there would be large difference in the weights at different iterations. This could be avoided if $\mathbf{u}_i$ is divided by its norm. A small positive value ϵ is added so that the algorithm does not breakdown in the situation when $\|\mathbf{u}_i\|^2 \approx 0$.

B.2 RLS Algorithm

In the previous subsection it was seen that the NLMS and LMS equations where an approximate iterative technique to minimize the mean square error. The RLS algorithm minimizes the Least Square (LS) error. Minimizing the least square cost is approximately equal to minimizing the mean square error when we have large number of observations. Assume that at time N where N is very large we approximate the mean square error as,

$$E\left[|d - \mathbf{u}\mathbf{w}|^2 \right] \approx \frac{1}{N} \sum_{i=0}^{N-1} |d(i) - \mathbf{u}_i \mathbf{w}|^2 \tag{B.7}$$

The LS cost function is obtained by removing the scaling factor $\frac{1}{N}$. This cost function could be written in vector form as,

$$J(\mathbf{w}_N) = \|\mathbf{y}_N - \mathbf{H}_N\mathbf{w}_N\|^2 \tag{B.8}$$

where

$$\mathbf{y}_N = \begin{bmatrix} d(0) \\ \vdots \\ d(N-1) \end{bmatrix} \tag{B.9}$$

and

$$\mathbf{H}_N = \begin{bmatrix} \mathbf{u}_0 \\ \vdots \\ \mathbf{u}_{N-1} \end{bmatrix} \tag{B.10}$$

This is the standard LS problem and for an overdetermined system its solution is $\mathbf{w}_N = (\mathbf{H}_N^H\mathbf{H}_N)^{-1}\mathbf{H}_N^H\mathbf{y}_N$. It can be observed that when $\mathbf{H}_N$ is rank deficient the inverse of $H_N^H H_N$ does not exist and hence $\mathbf{w}_N$ cannot be obtained. In order to alleviate this problem the LS cost function is modified as,

$$J(\mathbf{w}) = \lambda^{(N+1)}\mathbf{w}^H\Pi\mathbf{w} + (\mathbf{y}_N - \mathbf{H}_N\mathbf{w})^H\mathbf{\Lambda}_N(\mathbf{y}_N - \mathbf{H}_N\mathbf{w}) \tag{B.11}$$

where $\Lambda_N = diag\left\{\lambda^N, \lambda^{N-1}, \cdots, 1\right\}$ and λ is known as forgetting factor. The term $\mathbf{\Pi}$ is the regularization parameter.The cost function in (B.11) is known as exponentially weighted regularized least square function. Usually forgetting factor is a value less than 1. So in (B.11) it can be seen that as N increases the effect of regularization decreases. The solution for this optimization problem is,

$$\mathbf{w}_N = (\lambda^{N+1}\mathbf{\Pi} + \mathbf{H}_N^H\Lambda_N\mathbf{H}_N)^{-1}\mathbf{H}_N^H\Lambda_N\mathbf{y}_N \tag{B.12}$$

It can be seen in (B.12) that the inverse of the matrix always exist due to the presence of the regularization parameter which is a positive definite matrix. The RLS algorithm can be used to iteratively solve the linear equation in (B.12). Thus the complexity inherent in the matrix inversion is reduced.

Appendix C

Acronyms

AOFDM	-	Adaptive Orthogonal Frequency Division Multiplexing
BER	-	Bit Error Rate
BFAM 2D-RLS	-	Block Fast Array Multichannel 2D Recursive Least Square
DDCE	-	Decision Directed Channel Estimation
FAM 2D-RLS	-	Fast Array Multichannel 2D Recursive Least Square
LMS	-	Least Mean Square
MIMO	-	Multiple Input Multiple Output
MSE	-	Mean Square Error
NLMS	-	Normalized Least Mean Square
OFDM	-	Orthogonal Frequency Division Multiplexing
RLS	-	Recursive Least Square
SNR	-	Signal to Noise Ratio

Appendix D

Notations

In this thesis bold face capital letters are used for matrices and bold face lower case letters are used to represent vectors. Scalars are represented by normal font lower case letters. Conjugate transpose is represented by $(\cdot)^H$ and $(\cdot)^T$ is used to represent transpose operation. The expectation operator, trace and Euclidean norm are represented by $E[\cdot],\ tr\{\cdot\},\ \|\cdot\|$ respectively. We use $diag\{\cdot\}$ to represent both a diagonal matrix and the vector containing the diagonal elements of a matrix. Hadamard product is defined by $\odot$.

Bibliography

[1] A.Goldsmith,"*Wireless Communication*", Cambridge,2002.

[2] L.Hanzo, Y.Akhtman, L.Wang and M.Jiang,"*MIMO-OFDM for LTE, WIFI and WIMAX*", Wiley, 2011.

[3] S.W. Peters, A.Y. Panah, K.T.Truong and R.W. Heath Jr., "Relay architectures for 3GPP LTE-advances", in *EURASHIP Journal on wireless comm. and networking*, Vol.2009, May 2009.

[4] K.J.Ray, A.K.Sadek, W.Su and A.Kwasinski, "*Cooperative Communications and Networking*", Cambridge, 2008.

[5] P. Herhold, E. Zemmermann and G. Fettweis, " On the performance of cooperative relaying protocols in wireless networks", in *European transactions on telecommunications*", 2005.

[6] T. Unger, "*Multiple-antenna two-hop relaying for bi-directional transmission in wireless communication systems*",PhD thesis, June 2009.

[7] S.K. Jayaweera," Virtual MIMO-based cooperative communication for energy-constrained wireless sensor networks", in *IEEE trans. wireless communication*,Vol.5, May 2006.

[8] B. Rankov and A. Wittneben,"Spectral Efficient Protocols for Half-duplex Fading Relay Channels", in *IEEE Journal on Selected Areas in Communications*, Feb. 2007.

[9] F. Gao, R.Zhang and Y.C. Liang, "Channel estimation for OFDM modulated two-way relay networks", in *IEEE trans. on signal proc.*, vol. 57, no. 11, pp. 4443-4455, Nov. 2009.

[10] Y.(G.)Li, L.J.Cimini, N.R.Sollenberger,"Robust channel estimation for OFDM systems with rapid dispersive fading channels", in *IEEE trans.Commun.*,vol.46, no.7, pp.902-915, July 1998.

[11] P. Hoeher, S. Kaiser, and P. Robertson, "Two-dimensional pilot-symbol aided channel estimation by Wiener filtering", in *Proc. 1997 IEEE Int.Conf. Acoustics, Speech, and Signal Processing*, Munich, Germany, pp. 18451848, Apr.1997.

[12] L.Scharf, "*Statistical Signal Processing: Detection, Estimation, and Time Series Analysis*",Prentice Hall, 1991.

[13] X.Hou, S.Li, C.Yin and G.Yue,"Two-dimensional recursive least square adaptive channel estimation for OFDM systems", in *Int. Conf. on Wireless Commun,Networking and Mobile Computing*, pp.232-236, 2005.

[14] A.Soni, T.Sharma and V.Chakka,"Inverse QR 2D-RLS Adaptive Channel Estimation for OFDM Systems", *IEICE Transactions*, pp.2822-2825, 2010.

[15] A.Joy and V.Chakka,"Fast array multichannel 2D-RLS based OFDM channel estimator", submitted to *Ciruits, systems and signal processing*, Springer, 2012

[16] V.Mignone and A.Morello,"CD3-OFDM:A novel demodulation scheme for fixed and mobile receivers",*IEEE trans.commun.*, vol.44, pp.1144-1151, Sept 1996.

[17] A.Joy and V.Chakka, "Low Complexity 2D Adaptive Channel Estimation for OFDM Based Two-Way Relay Systems", submitted to *ICC 2013*, Hungary, 2013.

[18] G.L.Stuber *et al*,"Broadband MIMO-OFDM wireless communications",in *proc. of IEEE*, vol.92, Feb 2004.

[19] J.M.Cioffi,"A multicarrier primer", *ANSI T1E1. Committee Contribution*, 1991.

[20] J.Campello,"Practical bit loading for DMT",in *proc. ICC'99*, vol.2, 1999.

[21] L.Zheng, G.L.Stuber, D.Wang and Y.Cao," Optimal Bit and Power Direct Allocation Algorithm in OFDM Systems Based on the Regularity of Greedy Allocation Result", *IEEE. trans. on wireless comm.*, vol.11, sept. 2012.

[22] N.Y. Ermolova and B. Makarevitch, "On subchannel inversion for improvement of power efficieny in OFDM based systems", *IEE Communications*, pp. 1263-1266, Jan.2007.

[23] S. Nader and M. Afrasiabi, "Simple bit loading algorithm for OFDM based systems", *IEE Communications*, pp.312-316, Sept.2006.

[24] A.H.Sayed, *Fundamentals of Adaptive Filtering*, New York:Wiley, 2003.

[25] T.Unger and A.Klein,"Linear transceive filters for relay stations with multiple antennas in the Two-way relay channel", in *proc. of* 16^{th} *IST mobile and wireless communication summit*, Budapest, Hungary, July 2007.

[26] A.Papoulis, *Probability, Random Variables and Stochastic Processes*, Mcgraw-Hill College, Feb.1991.

[27] M.Joham, W.Utschick and J.A.Nossek, "Linear transmit processing in MIMO communications systems",in *IEEE tran. on signal Proc.*, vol.53,Aug.2005.

[28] S.Boyd and L.Vandenberghe,*Convex Optimization*, Cambridge, 2004.

[29] D.S.Watkins,"*Fundamentals of Matrix Computations*",New York:Wiley, 2002.

[30] A.Czylwik,"Adaptive OFDM for wideband radio channels", in *GLOBECOM'96*,London, 1996.

[31] X.Hou, S.Li, D.Liu, C.Yin and G.Yue,"On two-dimensional adaptive channel estimation in OFDM systems", in *proc.IEEE Vehicular tech. conf*, Los Angeles, Vol.1, pp.498-502, Sept. 2004.

[32] L.J. Cimini,Jr.,"Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing",*IEEE trans. Commun.*, vol.COM-33, pp.665-675, July 1985.

[33] M.K.Ozdemir and H.Arslan,"Channel estimation for wireless OFDM systems",*IEEE commun. survey and tutorials*, vol.9, pp.18-48, Jul 2007.

[34] J.-J. Van de Beek, O.Edfors, M.Sandell, S.K.Wilson and P.O.Börjesson,"On channel estimation in OFDM systems",in *Proc.* 45^{th} *IEEE Vehicular technology Conf.*, Chicago, IL, pp.915-819, July 1995.

[35] H.Minn and V.K.Bhargava,"An investigation into time-domain approah for OFDM channel estimation", in *Proc. IEEE trans. on broadcasting*, vol.68, pp.240-248, Dec.2000.

[36] Z.Tang, R.C.Cannizzaro,G.Leus,P.Banelli,"Pilot-Assisted Time-Varying Channel Estimation for OFDM Systems ", in *IEEE trans. signal proc.*, vol.55, pp. 2226-2238, April 2007.

[37] J.I.Montojo, L.B.Milstein,"Channel estimation for non-ideal OFDM systems", in *IEEE trans. commun.*, vol.58, pp. 146-156, Jan 2010.

[38] S.Coleri, M.Ergen, A.puri, A.Bahai,"Channel estimation techniques based on pilot arrangement in OFDM systems", in *IEEE trans. broadcasting*,vol.48, pp. 223-229, Nov.2002.

[39] B.Jiang, H.Wang, X.Gao, S.Jin, K.Wong," Preamble-Based Channel Estimation for Amplify-and-Forward OFDM Relay Networks",in *proc. of GLOBECOM 2009*,pp. 1-5, Mar.2010.

[40] D.Schafhuber, G.Matz and F. Hlawatsch,"Adaptive wiener filters for time varying channel estimation in wireless OFDM systems",in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.*, vol. 4, pp.688691, Apr.2003.

[41] T.Natori, N.Tanabe, H.Matsue, T.Furukawa,"Practical OFDM channel estimation method with Kalman filter theory ",in *proc. IEEE DSP/SPE 2011*,pp.18-23, Mar.2011.

[42] K.Fazel and S.Kaiser, "*Multi-Carrier and Spread Spectrum Systems: From OFDM and MC-CDMA to LTE and WiMAX* ", Wiley, 2008.

[43] D.C.Chang and H.C.Chiu,"A stabilized multichannel fast RLS algorithm for adaptive transmultiplexer receivers",*Circuits Syst. Signal Proc.*, vol.28,pp.845-867, Sept. 2009.

[44] R.West, T.moon and J.Gunther,"A novel block fast array RLS algorithm applied to linear flight strip-map SAR imaging",in *proc. ASILOMAR 2010*, pp.979-983, Nov. 2010.

[45] J.H.Husøy and M.S.E.Abadi,"Unified approach to adaptive filters and their performance",in *IET signal processing*, pp.97-109, 2008.

[46] Y.Saad,"*Iterative methods for sparse linear systems*", SIAM, 2003.

[47] L.F.Richardson, "The approximate arithmetical solution by finite difference of physical problems involving differential equations with an application to the stress to a masonry dam", *philos. trans. R.soc*,pp.307-357, 1910.

[48] K.Chen, "*Matrix Preconditioning techniques and applications*", Cambridge, 2005.

[49] S. Berger, T. Unger, M. Kuhn, A. Klein and A.Wittneben," Recent Advances in Amplify-and-Forward Two-Hop Relaying", in *IEEE Commuunication Magazine*, issue:7, pp. 50-56, July 2009.

[50] H. Ju, E. Oh and D. Hong," Catching Resource-Devouring Worms in Next-Generation Wireless Relay Systems: Two-Way Relay and Full-Duplex Relay", in *IEEE Communication Magazine*, issue:9, pp.58-65, Sept. 2009.

[51] K. Wesolowski, " Application of MIMO and network coding in two-way relaying applied in LTE", in *proc. IEEE* 21^{st} *inernational symposium on personal indoor and mobile radio communications*, pp. 619-624, Sept. 2010.

[52] G.wang,F.Gao,W.Chen and C.Tellambura, "Channel estimation and training design for two-way relay networks in time selective fading environments", in *IEEE trans. on wireless comm.*, vol.10, no.8, Aug 2011.

[53] T.H.Pham, Y.c.Liang, H.K.Garg, A.Nallanathan, “Joint channel estimation and data detection for MIMO-OFDM two-way relay networks”, in *proc. IEEE GLOBECOM 2010*, Miami, Dec 2010.

[54] A.Joy and V. Chakka, “ Performance comparison of LMS/NLMS based transceiver filters for MIMO two-way relaying scheme”,in *proc. ICCSP 2011*, Kerala, Feb. 2011.

[55] A.Joy and V.Chakka, “An IQR-RLS based transceiver filter for MIMO two-way relaying in frequency selective environment”, in *proc. UKIWCWS 2010*, New Delhi, Dec.2010.

[56] A.Joy and V.Chakka,“An Affine projection algorithm based transceiver filter for MIMO two-way relaying scheme”, in *proc. wireless VITAEE 2011*, Chennai, Mar. 2011.

[57] Won Gi Jeon, Kyung Hyun Paik, and Yong Soo Cho,“An efficient channel estimation technique for OFDM systems with transmitter diversity”, in *proc of PIMRC00*, vol.2, pp. 1246 -1250, Sept. 2000.

[58] I. Kalet, “The multitone channel”, in *IEEE trans. on communications*, vol.37, Feb 1989.

[59] J.A.C. Bingham,“Multicarrier modulation for data transmission: an idea whose time has come ”, in *IEEE communication magazine*, vol.28, pp. 5-14, May 1990.

[60] P.S. Chow, J.M.Cioffi and J.A.C.Bingham, “ A practical discrete multitone loading algorithm for data transmission over spectrally shaped channels”, in *IEEE trans. on communications*, vol.43, April 1995.

[61] A. Czylwik, “Adaptive OFDM for wideband radio channels”, in *proc. of GLOBECOM’96*, PP.713-718, Nov.1996.

[62] N.Yi, Y.Ma and R.Tafazolli, “Bit and power loading for OFDM with an amplify-and-forward cooperative relay”, in *proc. of PIMRC 2008*, PP.1-5, Sept.2008.

[63] O.Amin and M.Uysal, “Optimal bit and power loading for amplify and forward cooperative OFDM systems”, in *IEEE trans. on wireless comm.*, vol.10, March 2011.

[64] B.Gui and L.J.Cimini Jr., “Bit loading algorithms for cooperative OFDM systems”, in *EURASIP journal on wireless comm. and nerworking*,vol.2008, Sept. 2007.

[65] Y.Zhao, X.Wang and S.Li, “Joint adaptive modulation scheme for two-way relay channel”, in *IEE electronic letters*, vol.47, May 2011.

[66] S.Ye, R.S. Blum and L.J.Cimini Jr., “Adaptive OFDM systems with imperfect channel state information”, in *IEEE trans. on wireless comm.*, vol.5, Nov.2006.

[67] A.M. Wyglinski, F.Labeau and P.Kabal, "Effects of imperfect subcarrier SNR information on adaptive bit loading algorithms for multicarrier systems", in *in proc. GLOBECOM'04*, vol.6, 2004.

[68] C.Liu, D.Bielefeld and R.Mathar, "Optimal rate and power allocation for OFDM in the presence of channel uncertainity", in *proc. of PIMRC 2008*, PP.1-5, Sept.2008.

[69] S.Zhang, F.Gao, C.Pei, "Optimal Training Design for Individual Channel Estimation in Two-Way Relay Networks", in *IEEE trans. on signal proc.*, vol.60,Sept.2012.

[70] C.Assimakopoulos and F.N.Pavlidou, "Unified models for adaptive OFDM systems when QAM or PSK modulation is applied", in *European transactions on telecommunication*, pp.777-790, Feb.2007.